Name:	

[5] 1. True or False ( $\emptyset$  represents the empty set):  $\{1, 4\} \subset \{1, 2, 3, 4\}$ 

(b) 
$$\{b\} \in \{a, \{b, c\}\}$$

(a)

(c) 
$$\{a, \{b, c\}\} \subseteq \{a, b, c, d\}$$

(d) 
$$\emptyset \subseteq A$$
 for any set  $A$ 

(e) 
$$A \cup \emptyset = \emptyset$$
 for any set  $A$ 

[2] 2. List all the elements in the set 
$$A = \{x \mid x = 7k + 1 \text{ where } k = 0, 3, 5\}.$$

[5] 3. Use Venn diagrams to determine if 
$$(A \cap B) - C = (A - C) \cap (B - C)$$
 is true or not.

[6] 4. Given the sets  $U = \{a, b, c, d, e, f, g, h, i, j\}$ ,  $A = \{a, b, c, d, f, i\}$ ,  $B = \{d, e, g\}$  and  $C = \{b, d, g\}$ , find:
(a)  $A - (B \cap C)$ 

(b)  $\overline{A} \cup B$ 

(c)  $C \times B$ 

[7] 5. Use a Venn diagram to find (i)  $n(\overline{A \cup B \cup C})$ , (ii)  $n(A \cap (B \cup C))$  and (iii)  $n((A \cup B) - C)$  if  $n(A \cap B \cap C) = 5$ ,  $n(A \cap C) = 12$ ,  $n(B \cap C) = 6$ , n(A) = 27,  $n(A - (B \cup C)) = 13$ , n(B) = 12, n(C) = 28 and n(U) = 55.

[5] 6. The university's drama department is planning to put on the play "Game of Cones" at the university theatre. If they charge \$5.00 per person, the theatre will be filled to capacity but if they increase the price by \$2.00 per ticket, 15 less tickets will be sold but the total revenue will increase by \$317. Find the capacity of the school theatre.

- [4] 7. Given the sets  $A = \{1, 2, 3, 4, 5, 6, 7\}$  and  $B = \{a, b, c, d, k, u, v\}$ , (a) find a **one-to-one correspondence** between the two sets.
  - (b) How many different one-to-one correspondences are there between A and B if both 1 and 2 must correspond to consonants (non-vowels)?

[4] 8. How many different 4-digit numbers can be formed using the digits: 1, 3, 4, 5, 7, 8, 9 if the first digit must be even and the second must be odd and repeated digits are **not** allowed?

[3] 9. Determine if it is possible to create a  $3 \times 3$  magic square with the numbers: 3, 4, 7, 8, 9, 11, 12, 14, 17. Justify your answer.

[6] 10. For each of the following sequences, (i) <u>identify</u> the sequence, (ii) find  $a_{123}$  and (iii) give a <u>direct formula</u> for the  $n^{th}$  term of the sequence: (a) 2, 9, 16, 23, .....

(b) 4, 12, 36, 108, .....

[3] 11. Given the recursive formula:  $a_n = 3a_{n-1} + 5a_{n-2}$  with  $a_1 = 3$  and  $a_2 = 2$ , find  $a_3$  and  $a_4$ .

[BONUS QUESTION] Evaluate and simplify  $\frac{8-1-7}{3-3}$ . [2]





Name: \_\_\_\_\_

[4] 1. Name the **property of whole number operations** demonstrated by each of the following statements:

(a) 
$$(a \times b) \times c = (b \times a) \times c$$

(b) 
$$(2x+y) = (2x+y) \cdot 1$$

(c) 
$$(a+b)+(c+d)=a+(b+(c+d))$$

- (d) For all  $x, y \in W$ , x + y is also a whole number
- [4] 2. Show that the equation a + (b + c) = (c + a) + b is true. Give a property of addition to justify each step in your argument.

- [5] 3. An operation  $\oplus$  is defined on the set  $S = \{d, o, b\}$  as shown in the table on the right. For example,  $d \oplus o = o$  and  $b \oplus d = d$  etc.
  - (a) Is S closed with respect to  $\oplus$ ? Explain.

$\oplus$	d	0	b
d	b	0	d
0	b	0	0
<i>b</i> .	d	0	b

- (b) Is  $\oplus$  commutative on S? Explain.
- (c) Is there an identity element for  $\oplus$  on S? If yes, what is it?

[3] 4.	Write $C0DE_{16}$ in expanded form and convert it to base 10.
[9] 5.	Perform the following conversions:  (a) Write 487,943,294 in Roman numerals.
	(b) Write < < < ▼▼▼▼▼▼    A
	(c) Write 124,000 in Mayan numerals.
[4] 6.	Use the successive division algorithm discussed in class to convert 2019 to a base 6 number.

[8] 7. Perform each of the following operations using the given bases:

(a) 
$$254_7 + 656_7$$

(b) 
$$10010_2$$
  $- 111_2$ 

(c) 
$$BCA \, 8_{16} + DCD_{16}$$

(d) 
$$FCC7_{16} - 1F9_{16}$$

[3] 8. Use the lattice algorithm to perform the following operation:  $654_8 + 726_8$ .

[3] 9. Use the scratch algorithm to find the sum:

$$\begin{array}{r}
 35_7 \\
 26_7 \\
 14_7 \\
 53_7 \\
 + 32_7
 \end{array}$$

[3] 10. Name the 3 different subtraction models discussed in class.

[4] 11.Use the distributive property to factor the following expressions completely:

(a) 
$$28x^4y^2 + 35x^3y^5 =$$

(b) 
$$15(x+1)^5 + 5(x+1)^2 =$$

[BONUS QUESTION]

[2] Explain why for any non-zero number  $a, a \div 0$  is undefined.

Name:	

[3] 1. Use <u>lattice algorithm</u> to perform the following operation:

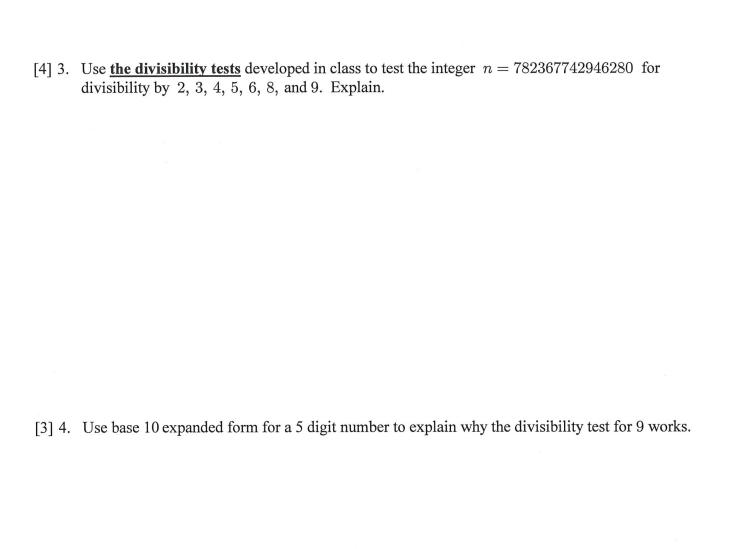
$$546_7 \times 435_7 =$$

[9] 2. Perform the following operations using the **standard algorithms** in the given bases:

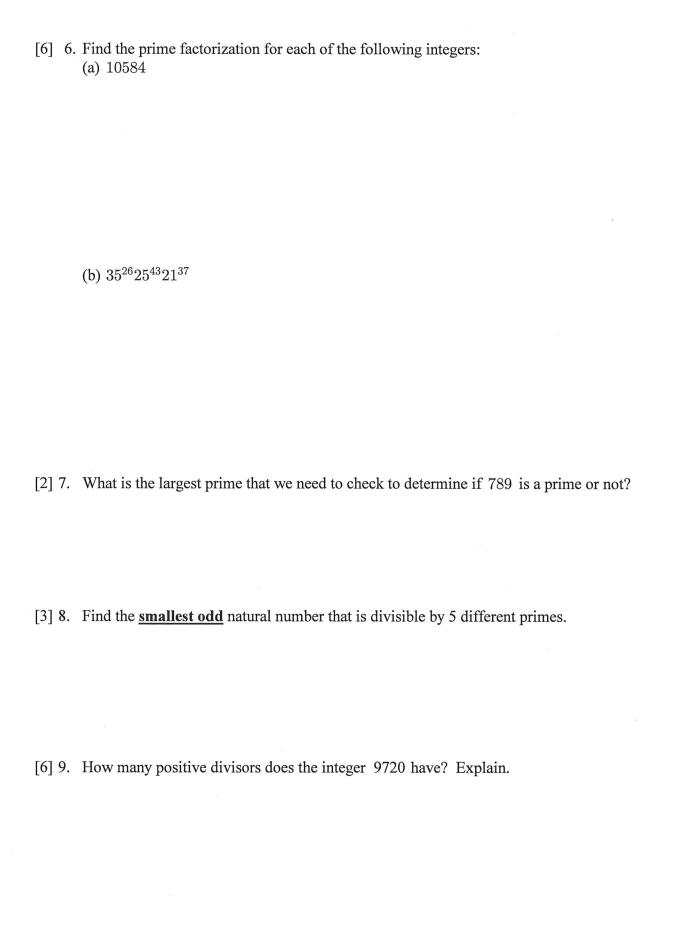
(a) 
$$523_6 \times 43_6 =$$

(b) 
$$111010_2 \div 110_2 =$$

(c) 
$$652_8 \div 7_8 =$$



[4] 5. Let a, b and c be integers with  $a \neq 0$ , use **the definition of divisibility** to prove that if a|b and a|c then  $a^2|5bc$ .



[4] 10. Find GCD(a, b) and LCM(a, b) if  $a = 2^{52}3^{73}7^{49}$  and  $b = 3^{54}7^{55}11^{29}$ .

[6] 11. Use the <u>Euclidean Algorithm</u> to find GCD(18375, 9408), then use the GCD to find LCM(18375, 9408).

## [BONUS QUESTION]

[2] What is the smallest natural number that has exactly 13 different positive divisors?