Ex 6.4

52/ no. of ways 5 people can have 5 different birthdays

$= P(365, 5)$

$= 365 \times 364 \times 363 \times 362 \times 361$

$= \underline{6.3026 \times 10^{12}}$

53/ (a) no. of different ways to arrange the letters in "SUNDAY"

$= 6!$

$= \underline{720}$

(b) no. of different ways to arrange the letters if S must come first

$= 1 \times 5!$

$= \underline{120}$

(c) no. of different ways if S comes first and Y be the last

$= 1 \times 4! \times 1$

$= \underline{24}$

Ex 6.4

1. No. of 5 letter strings if repeated letters are ~~not~~ allowed

$= 26 \times 25 \times 24 \times 23 \times 22$

$= \underline{7,893,600}$

No. of 5 letter strings if repeated letters are allowed

$= 26 \times 26 \times 26 \times 26 \times 26$

$= \underline{11,881,376}$

* 54. (a) If books of the same language must be grouped together then

no. of ways to arrange them on a shelf

$= \begin{pmatrix}\text{no. of ways to}\\ \text{arrange the}\\ \text{2 languages}\end{pmatrix} \times \begin{pmatrix}\text{no. of ways to}\\ \text{arrange the}\\ \text{French books}\end{pmatrix} \times \begin{pmatrix}\text{no. of ways to}\\ \text{arrange the}\\ \text{Spanish books}\end{pmatrix}$

$= 1 \times 5! \times 5!$

$= 1 \times 120 \times 120$

$= \underline{14400}$

(b) If French and Spanish books must alternate then

no. of ways to arrange them on a shelf

$= \begin{pmatrix}\text{no. of ways to}\\ \text{alternate the}\\ \text{languages}\end{pmatrix} \times \begin{pmatrix}\text{no. of ways to}\\ \text{arrange the}\\ \text{French books}\end{pmatrix} \times \begin{pmatrix}\text{no. of ways to}\\ \text{arrange the}\\ \text{Spanish books}\end{pmatrix}$

$= 1 \times 5! \times 5!$

$= 1 \times 120 \times 120$

$= \underline{14400}$

56/ no. of ways to assign 8 printers to 4 outputs

$$= \begin{pmatrix}\text{no. of ways to}\\ \text{assign a printer}\\ \text{to output \#1}\end{pmatrix} \times \begin{pmatrix}\text{no. of ways to}\\ \text{assign a printer}\\ \text{to output \#2}\end{pmatrix} \times \cdots \times \begin{pmatrix}\text{no. of ways to}\\ \text{assign a printer}\\ \text{to output \#4}\end{pmatrix}$$

$$= 8 \times 7 \times 6 \times 5$$

$$= \underline{1,680}$$

58/ no. of ways to fill 7 different positions

$$= \begin{pmatrix}\text{no. of ways}\\ \text{to fill the}\\ 1\text{st spot}\end{pmatrix} \times \begin{pmatrix}\text{no. of ways}\\ \text{to fill the}\\ 2\text{nd spot}\end{pmatrix} \times \cdots \times \begin{pmatrix}\text{no. of ways}\\ \text{to fill the}\\ 7\text{th spot}\end{pmatrix}$$

$$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$$

$$= \underline{604,800}$$

Ex 6.5

1. No. of different teams

$$= \begin{pmatrix}\text{no. of ways}\\ \text{to choose}\\ 2 \text{ forwards}\end{pmatrix} \times \begin{pmatrix}\text{no. of ways}\\ \text{to choose}\\ 2 \text{ guards}\end{pmatrix} \times \begin{pmatrix}\text{no. of ways}\\ \text{to choose}\\ 1 \text{ centre}\end{pmatrix}$$

$$= C(7,2) \times C(3,2) \times C(2,1)$$

$$= \frac{7!}{2!\,5!} \times \frac{3!}{2!\,1!} \times \frac{2!}{1!\,1!}$$

$$= 21 \times 3 \times 2$$

$$= \underline{126}$$

Ex 6.4

no. of ways to rank 8 candidates who apply for a job

= no. of ways to arrange 8 objects in a row

= 8!

= 40,320

Ex 6.5

23/ no. of 8-bit strings with exactly three 1's

= C(8, 3)

$= \frac{8!}{3!\,5!}$

= 56

24/ no. of 8-bit strings with exactly two 1's :

:= C(8, 2)

$= \frac{8!}{2!\,6!}$

= 28

Ex 6.5

27/ no. of 9-letter words formed using the letters in ECONOMICS

$$= \frac{9!}{1!2!2!1!1!1!1!}$$

(1E, 2C's, 2O's, 1N, 1M, 1I and 1S)

$$= 90,720$$

40/ (d) no. of samples containing exactly 4 errors

$$= C(5,4) \times C(53,4)$$

$$= \frac{5!}{4!\,1!} \times \frac{53!}{4!\,49!}$$

$$= 5 \times 292825$$

$$= 1,464,125$$

(e) no. of samples containing exactly 5 errors

$$= C(5,5) \times C(53,3)$$

$$= \frac{5!}{0!\,5!} \times \frac{53!}{3!\,50!}$$

$$= 1 \times 23,426$$

$$= 23426$$

(f) no. of samples containing at least one error

$$= C(58,8) - C(53,8)$$

$$= \frac{58!}{8!\,50!} - \frac{53!}{8!\,45!}$$

$$= 1916797311 - 886322710$$

$$= 1,030,474,601$$

(g) no. of samples with no errors

$$= C(53,8)$$

$$= 886322710$$

Ex 6.5

29/. no. of ways to arrange 3 red, 4 yellow and 5 blue bulbs

$$= \frac{12!}{3!\,4!\,5!}$$

$$= \underline{27,720}$$

34/. (a) no. of committees if a person can serve on any number of committees

$$= C(9,4) \times C(9,3) \times C(9,2)$$

$$= \frac{9!}{4!\,5!} \times \frac{9!}{3!\,6!} \cdot \frac{9!}{2!\,7!}$$

$$= 126 \times 84 \times 36$$

$$= \underline{381,024}$$

(b) no. of committees if no person can serve on more than one committee

$$= C(9,4) \times C(5,3) \times C(2,2)$$

$$= \frac{9!}{4!\,5!} \times \frac{5!}{3!\,2!} \times \frac{2!}{2!\,0!}$$

$$= 126 \times 10 \times 1$$

$$= \underline{1260}$$

Ex 6.5

33/. no. of ways to form teams of 3, 5 and 4 members

$$= \frac{12!}{3!\,5!\,4!}$$

$$= \underline{27,720}$$

Ex 6.5

34/ no. of committees with 2 boys and 3 girls

$= \begin{pmatrix} \text{no. of ways to choose} \\ \text{2 boys from 4 boys} \end{pmatrix} \times \begin{pmatrix} \text{no. of ways to choose} \\ \text{3 girls from 8 girls} \end{pmatrix}$

$= C(4,2) \times C(8,3)$

$= \frac{4!}{2!\,2!} \times \frac{8!}{3!\,5!}$

$= 6 \times 56$

$= \underline{336}$

50/ no. of committees with 5 senators and 4 representatives

$= \begin{pmatrix} \text{no. of ways to choose} \\ \text{5 senators from 100} \end{pmatrix} \times \begin{pmatrix} \text{no. of ways to choose} \\ \text{4 representatives from 435} \end{pmatrix}$

$= C(100,5) \times C(435,4)$

$= \frac{100!}{5!\,95!} \times \frac{435!}{4!\,431!}$

$= 75,287,520 \times 1,471,429,260$

$= \underline{1.107802 \times 10^{17}}$

54/ no. of different assortments

= no. of ways to choose 3 items from 8 items

$= C(8,3)$

$= \frac{8!}{3!\,5!}$

$= \underline{56}$