MATH 111

HW#9

1

Ex 6.1

58/ All possible subsets of $\{a, b, c, d\}$ are :

ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{b,c\}$
$\{b,d\}$, $\{c,d\}$, $\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$
$\{a,b,c,d\}$

All are proper subsets except $\{a,b,c,d\}$

26/ $U = \{1,2,3,4,5\}$, $A = \{3,5\}$, $B = \{1,2,3\}$, $C = \{2,3,4\}$

(a) $\overline{A} \cap \overline{B} = \{1,2,4\} \cap \{4,5\} = \underline{\{4\}}$

(c) $A \cup (B \cap C) = \{3,5\} \cup \{2,3\} = \underline{\{2,3,5\}}$

(e) $\overline{A \cap C} = \overline{\{3\}} = \underline{\{1,2,4,5\}}$

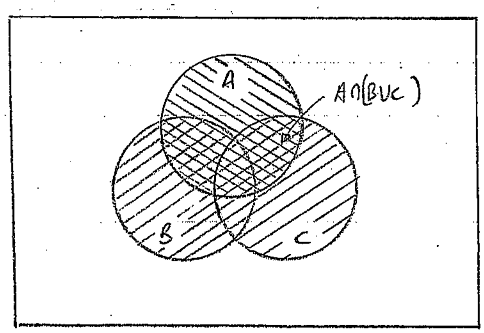
(g) $\overline{A} \cup \overline{B} = \{1,2,4\} \cup \{4,5\} = \underline{\{1,2,4,5\}}$

37/ $A \cap (B \cup C)$

$B \cup C$

A

$A \cap (B \cup C)$

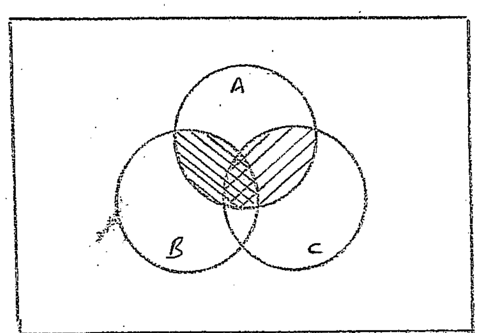


$(A \cap B) \cup (A \cap C)$

$A \cap C$

$A \cap B$

Any shaded area $(A \cap B) \cup (A \cap C)$



Since the two regions are identical
$\underline{A \cap (B \cup C) = (A \cap B) \cup (A \cap C)}$

Ex 6.1

50%. $\bar{M} \cap \bar{S}$ = set of all students who are female and do not smoke.

Ex 6.2

14%. Given that $n(A) = 10$, $n(A \cap B) = 5$ and $n(A \cup B) = 29$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\begin{aligned} n(B) &= n(A \cup B) + n(A \cap B) - n(A) \\ &= 29 + 5 - 10 \\ &= \underline{\underline{24}} \end{aligned}$$

Ex 6.2

16/. Let H be the set of 1^{st} year students who are taking history

M " " " " " " " " mathematics

then $c(H) = 350$, $c(M) = 300$, $c(H \cap M) = 270$

no. of students who are taking one or both

$= c(H \cup M)$

$= 350 + 300 - 270$

$= \underline{380}$

~~no. of students who are taking neither~~

$= c(\overline{H \cup M})$

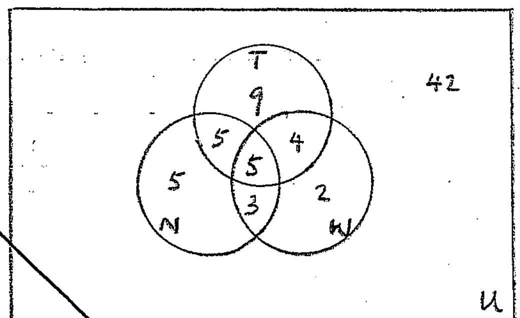
$= 1500 - c(H \cup M)$

$= \underline{1120}$

38/. Let T be the set of students who read Time

N " " " " " " Newsweek

W " " " " " " U.S. News & World Report.



(a) $c(\overline{T \cup N \cup W}) = \underline{42}$

(b) $c(T \cap \bar{N} \cap \bar{W}) = \underline{9}$

(c) $c(\bar{T} \cap N \cap \bar{W}) = \underline{5}$

(d) $c(\bar{T} \cap \bar{N} \cap W) = \underline{2}$

(e) $c(\overline{T \cup N}) = 42 + 2 = \underline{44}$

(f) $c(T \cup N) = 9+4+5+5+5+3 = \underline{31}$

Ex 6.2

39/ Let $U = \{\text{all cars sold in July}\}$

$H = \{\text{all cars sold with heated seats}\}$

$G = \{\text{all cars sold with GPS}\}$

$S = \{\text{all cars sold with satellite radio}\}$

then $n(H) = 90$, $n(G) = 100$, $n(S) = 75$

$n(H \cap G \cap S) = 5$

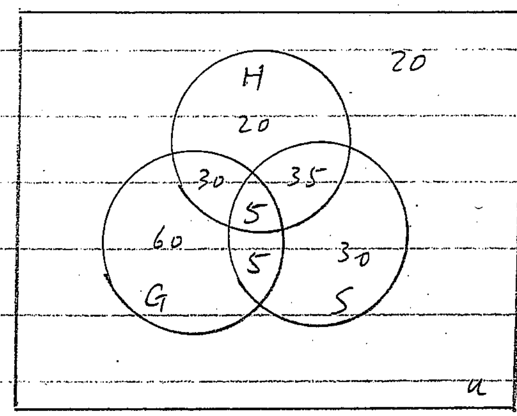
$n(\bar{H} \cap \bar{G} \cap \bar{S}) = 20$

$n(H \cap \bar{G} \cap \bar{S}) = 20$

$n(\bar{H} \cap G \cap \bar{S}) = 60$

$n(\bar{H} \cap \bar{G} \cap S) = 30$

$n(G \cap S) = 10$



(a) no. of cars with both satellite radio and heated seats

$= n(H \cap S)$

$= 35 + 5 = \underline{\underline{40}}$

(b) no. of cars with both GPS and heated seats

$= n(H \cap G)$

$= 30 + 5 = \underline{\underline{35}}$

(c) no. of cars with neither satellite radio nor GPS

$= n(\bar{S} \cap \bar{G})$

$= 20 + 20 = \underline{\underline{40}}$

(d) no. of cars sold in July

$= n(u)$

$= \underline{\underline{205}}$

(e) no. of cars with GPS or heated seats or both

$= n(G \cup H) = \underline{\underline{155}}$

8

Ex 6.3

~~1/ no. of ways to arrange 3 cards~~

~~$= \binom{\text{no. of ways to}}{\text{choose the 1st card}} \times \binom{\text{no. of ways to}}{\text{choose the 2nd card}} \times \binom{\text{no. of ways to}}{\text{choose the 3rd}}$~~

~~$= 6 \times 5 \times 4$~~

~~$= \underline{\underline{120}}$~~

31/ (a) no. of 7-digit numbers with no repeated digits (lead 0 allowed)

$= 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$

$= \underline{\underline{604,800}}$

(b) no. of 7-digit numbers with no repeated digits and no leading 0

$= 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$

$= \underline{\underline{544,320}}$

(c) no. of 7-digit numbers with repeated digits (lead 0 allowed)

$= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

$= \underline{\underline{10,000,000}}$

34/ no. of ways to answer the test

$= \underbrace{4 \times 4 \times \cdots \times 4}_{10 \text{ multiple choice questions}} \times \underbrace{2 \times 2 \times \cdots \times 2}_{15 \text{ T/F questions}}$

$= 4^{10} \times 2^{15}$

$= 2^{20} \times 2^{15}$

$= 2^{35}$

$= \underline{\underline{34,359,738,368}}$

Ex 6.3

33/. (a) no. of ways to arrange the 7 letters in "PROBLEM"
$= 7!$
$= \underline{5040}$

(b) no. of ways to arrange the 7 letters if "P" must come first
$= 1 \times 6!$
$= \underline{720}$

(c) no. of ways to arrange the 7 letters if "P" must come first and "M" must be last
$= 1 \times 5! \times 1$
$= \underline{120}$

35/. (a) no. of license plates if letters and digits may be repeated
$= 26 \times 26 \times 10 \times 10 \times 10 \times 10$
$= \underline{6,760,000}$

(b) no. of license plates if letters may be repeated but digits may not
$= 26 \times 26 \times 10 \times 9 \times 8 \times 7$
$= \underline{3,407,040}$

(c) no. of license plates with no repeated digits or letters
$= 26 \times 25 \times 10 \times 9 \times 8 \times 7$
$= \underline{3,276,000}$

Ex 6.3

40/ no. of different plates $= 24 \times 9 \times 10 \times 10 \times 10$

$= \underline{\underline{216,000}}$

46/ Number of 3-letter code words

(a) if no letter can be repeated is

$10 \times 9 \times 8$

$= \underline{\underline{720}}$

(b) if letters can be repeated is

$10 \times 10 \times 10$

$= \underline{\underline{1000}}$

(c) if adjacent letters cannot be the same

$= 10 \times 9 \times 9$

$= \underline{\underline{810}}$

37/ No. of distinguishable car types

$= \left(\text{no. of cars of type } A\right) + \left(\text{no. of cars of type } B\right) + \left(\text{no. of cars of type } C\right)$

$= (3 \times 2) + (3 \times 2) + (2 \times 2)$

$= \underline{\underline{16}}$