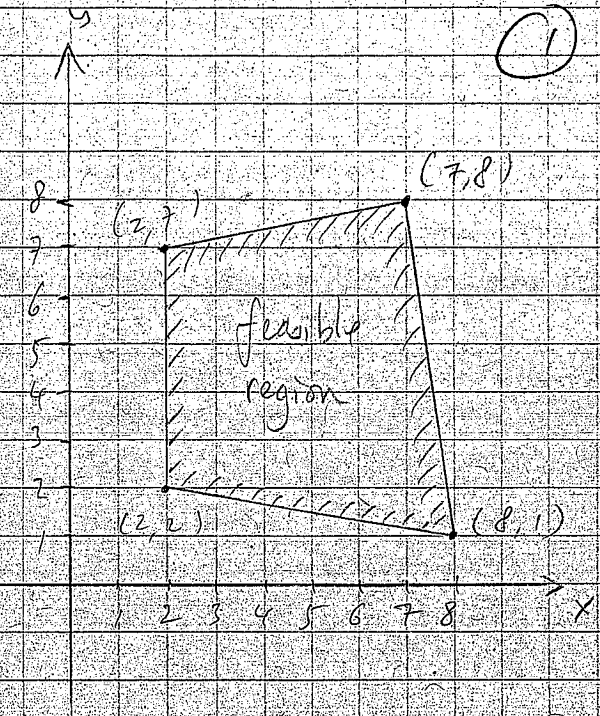
Ex 3.2

14/

$Z = x + 10y$

vertex	Z
$(2,2)$	$2+10(2)=22$
$(2,7)$	$2+10(7)=72$
$(7,8)$	$7+10(8)=87$
$(8,1)$	$8+10(1)=18$



The maximum value of z is 87 and the minimum is 18

/ max $Z = 5x + 7y$

subject to

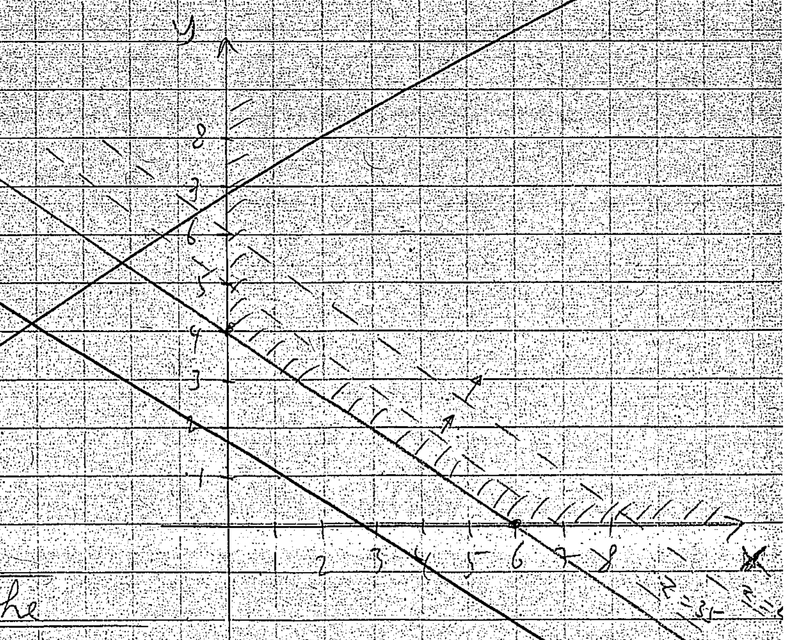
$2x + 3y \geq 12$

$x, y \geq 0$

Since the region is unbounded, graph

$35 = 5x + 7y$

$42 = 5x + 7y$



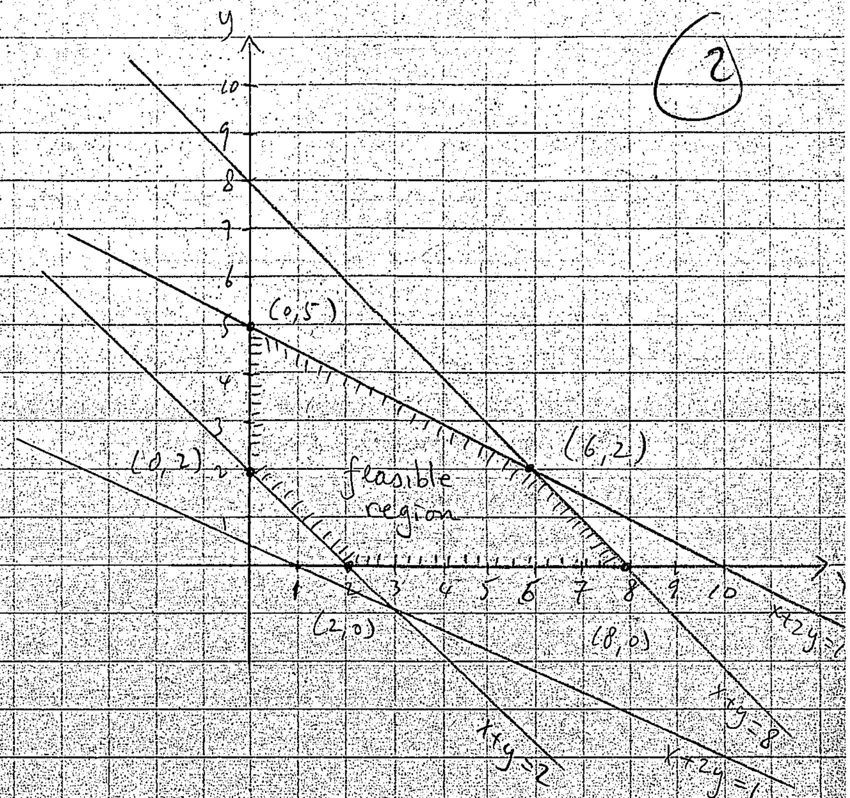
The z-line can be moved upward without bound, the LP is unbounded

Ex 3.2

26/ $\max\ z = 5x + 7y$

subject to

$$2 \le x + y$$
$$x + y \le 8$$
$$1 \le x + 2y$$
$$x + 2y \le 10$$
$$x, y \ge 0$$



Since the feasible region is bounded, there is an optimal solution

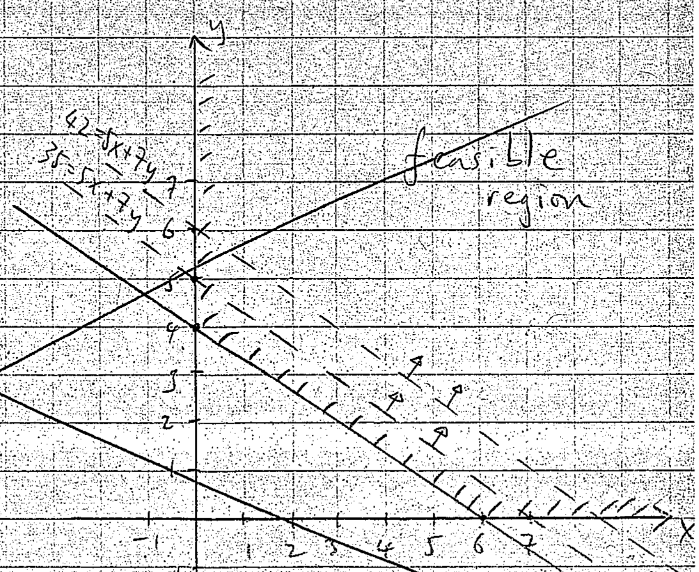
Vertices	z	
(0, 2)	$5(0) + 7(2) = 14$	
(0, 5)	$5(0) + 7(5) = 35$	
(6, 2)	$5(6) + 7(2) = 44$	← max
(8, 0)	$5(8) + 7(0) = 40$	
(2, 0)	$5(2) + 7(0) = 10$	

The optimal solution is $x^* = 6$, $y^* = 2$, $z^* = 44$

7/ $\max\ z = 5x + 7y$

subject to

$$x \ge 0$$
$$y \ge 0$$
$$2x + 3y \ge 12$$



Since the region is unbounded

graph $35 = 5x + 7y$

$42 = 5x + 7y$

The z-line can be moved upward without bound, the LP is unbounded

Ex 3.2

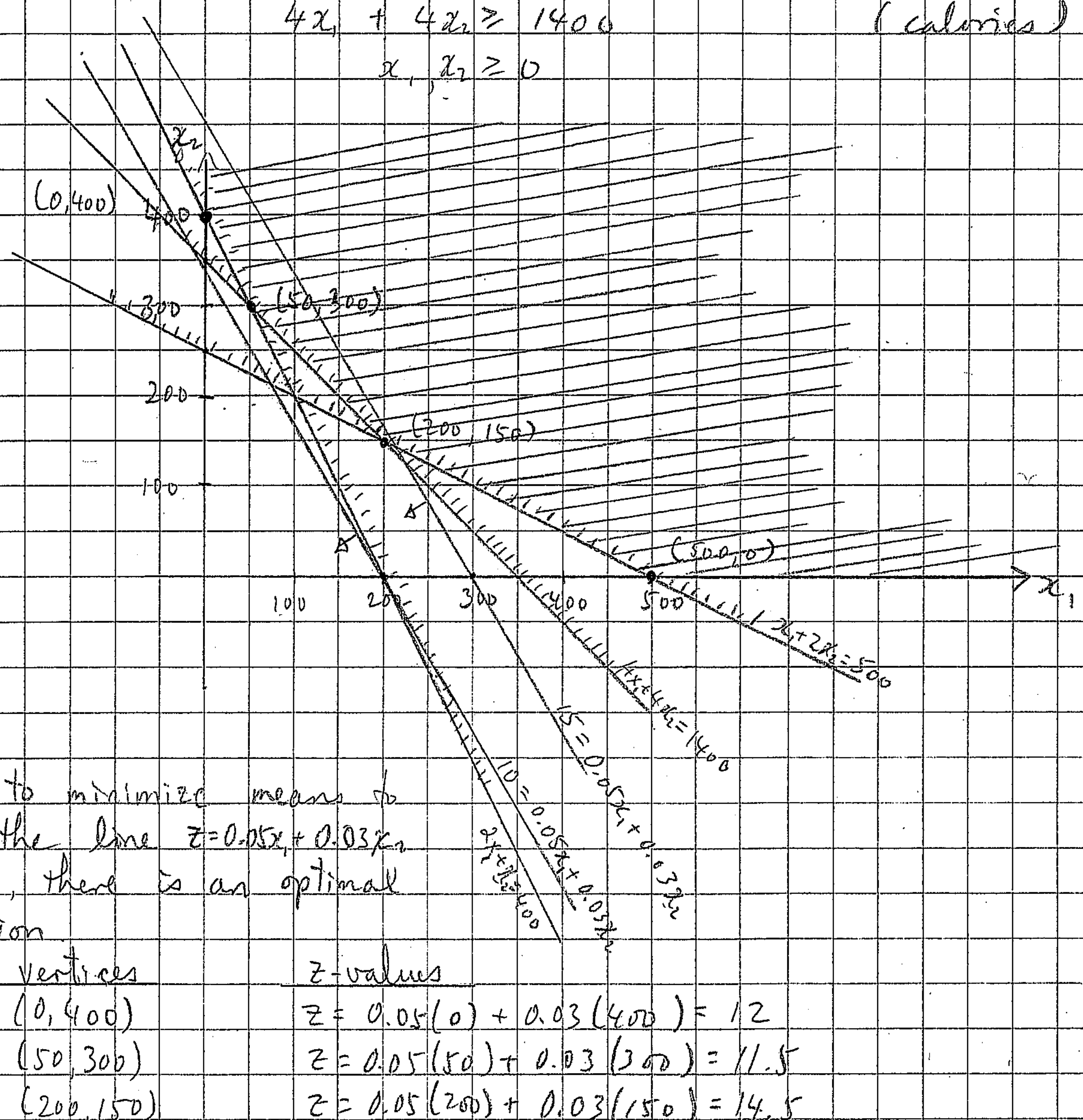
(4). Let x_1 be the amount of food F_1 in the diet
x_2 " " " " F_2 " " "

To objective is to

$$\text{Min } Z = 0.05x_1 + 0.03x_2 \qquad \text{(cost of diet)}$$

Subject to

$$2x_1 + x_2 \geq 400 \qquad \text{(vitamins)}$$
$$x_1 + 2x_2 \geq 500 \qquad \text{(minerals)}$$
$$4x_1 + 4x_2 \geq 1400 \qquad \text{(calories)}$$
$$x_1, x_2 \geq 0$$



Since to minimize means to move the line $Z = 0.05x_1 + 0.03x_2$ down, there is an optimal solution

vertices	Z-values
(0, 400)	$Z = 0.05(0) + 0.03(400) = 12$
(50, 300)	$Z = 0.05(50) + 0.03(300) = 11.5$
(200, 150)	$Z = 0.05(200) + 0.03(150) = 14.5$
(500, 0)	$Z = 0.05(500) + 0.03(0) = 25$

The optimal solution is $x_1^* = 50$, $x_2^* = 300$, $Z^* = 11.5$

78/ Let x_1 be the no. of rolls of high grade carpet manufactured
x_2 " " " " low grade " "

The objective is to

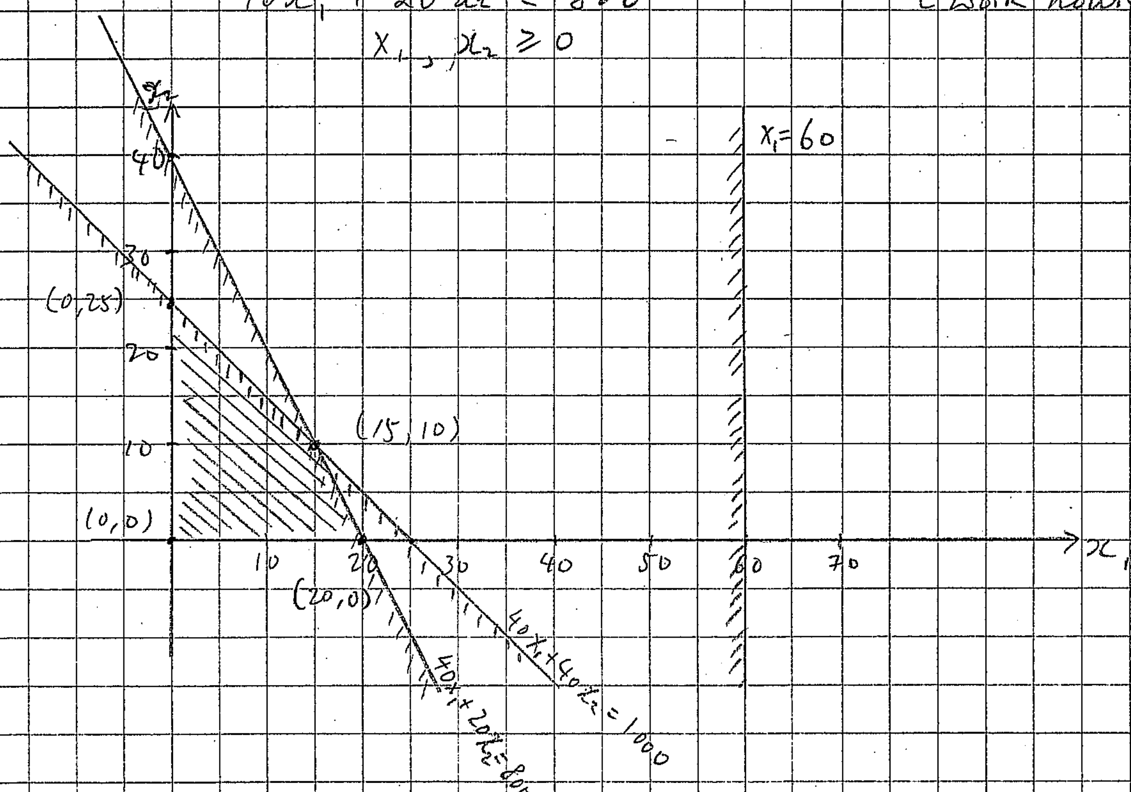
$$\text{Max } z = \underbrace{(500x_1 + 300x_2)}_{\text{revenue}} - \underbrace{(40\cdot 6\cdot x_1 + 20\cdot 6\cdot x_2)}_{\text{labour cost}} - \underbrace{5(20x_1) - 2(40x_1 + 40x}_{\text{material cost}}$$

$$= 500x_1 + 300x_2 - 240x_1 - 120x_2 - 100x_1 - 80x_1 - 80x_2$$

$$= 80x_1 + 100x_2$$

Subject to

$20x_1 \le 1200$		(wool)
$40x_1 + 40x_2 \le 1000$		(nylon)
$40x_1 + 20x_2 \le 800$		(work hours)
$x_1, x_2 \ge 0$		



Since the feasible region is bounded, there is an optimal solution

Vertices	z-values
(0, 25)	$z = 80(0) + 100(25) = 2500$
(15, 10)	$z = 80(15) + 100(10) = 2200$
(20, 0)	$z = 80(20) + 100(0) = 1600$
(0, 0)	$z = 80(0) + 100(0) = 0$

The optimal solution is $x_1^* = 0$, $x_2^* = 25$, $z^* = 2500$

Ex 4.2

32(a)

Categories	I	II	III	max. no. of hrs
time	$\frac{1}{2}$ hr	1 hr	2 hrs	4×10
fee	\$600	\$900	\$1200	

Let x_1 be the no of operations of type I
x_2 -- -- -- -- II
x_3 -- -- -- -- III

then we want to

$$\max \quad z = 600x_1 + 900x_2 + 1200x_3 \qquad \text{(revenue)}$$

subject to

$$x_1 + x_2 + x_3 \le 60 \quad \text{(total no. of operations)}$$
$$\tfrac{1}{2}x_1 + x_2 + 2x_3 \le 40 \quad \text{(no. of hrs)}$$
$$x_1, x_2, x_3 \ge 0$$

38(a)

	stocks	corporate bonds	municipal bonds
yield	10%	8%	6%

Let x_1 be the amt. invested in stocks
x_2 -- -- -- in corporate bonds
x_3 -- -- -- municipal bonds

then we need to

$$\max \quad z = 0.10x_1 + 0.08x_2 + 0.06x_3 \qquad \text{(return in one year)}$$

subject to

$$x_1 \le 45000$$
$$x_2 - x_3 \le 18000$$
$$x_1 + x_2 + x_3 \le 90000$$
$$x_1, x_2, x_3 \ge 0$$

Ex 4.2

40/(a)

	copper	lead	zinc	max. no. of hrs
oiling	2	2	1	10
mixing	2	3	1	11
separation	1	1	3	10
profit/unit	\$45	\$30	\$35	

Let x_1 be the no. of units of copper produced
x_2 — — — — lead ..
x_3 .. .. — .. zinc ..

then we need to

max $z = 45x_1 + 30x_2 + 35x_3$ (profit)

subject to

$$2x_1 + 2x_2 + x_3 \le 10 \quad \text{(oiling)}$$
$$2x_1 + 3x_2 + x_3 \le 11 \quad \text{(mixing)}$$
$$x_1 + x_2 + 3x_3 \le 10 \quad \text{(separation)}$$
$$x_1, x_2, x_3 \ge 0$$

Ex 4.2

4.2 (a)

	Sanding	Staining	Varnishing	profit
End table	8	10	4	\$15
Cocktail table	4	4	8	\$20
amount of Time available	6	6	6	

Let x_1 be the no. of end tables manufactured
x_2 be the no. of cocktail tables manufactured.

Then we need to

$$\max \quad z = 15x_1 + 20x_2$$

subject to

$$8x_1 + 4x_2 \le 360$$
$$10x_1 + 4x_2 \le 360$$
$$4x_1 + 8x_2 \le 360$$
$$x_1, x_2 \ge 0$$

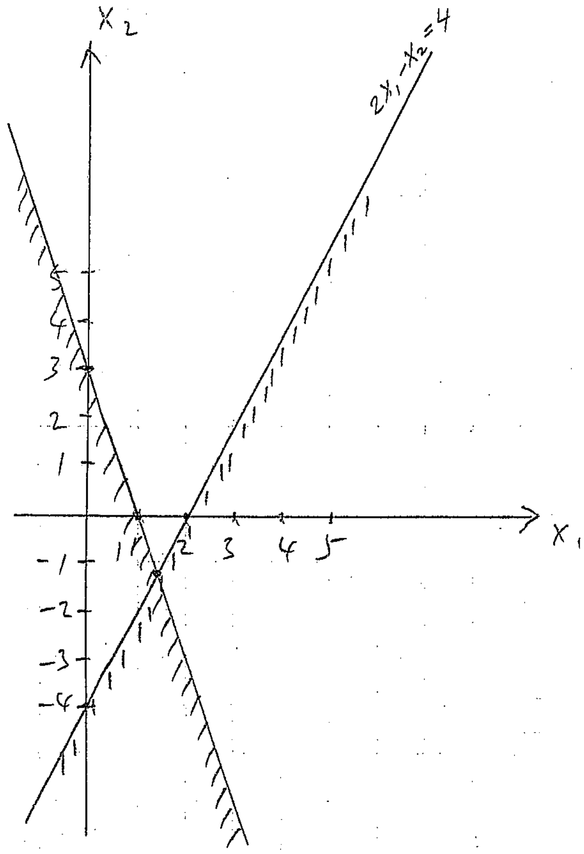
Extra

Solve the LP:

$$\min \quad z = x_1 + x_2$$

subject to

$$2x_1 - x_2 \geq 4$$
$$3x_1 + x_2 \leq 3$$
$$x_1, x_2 \geq 0$$



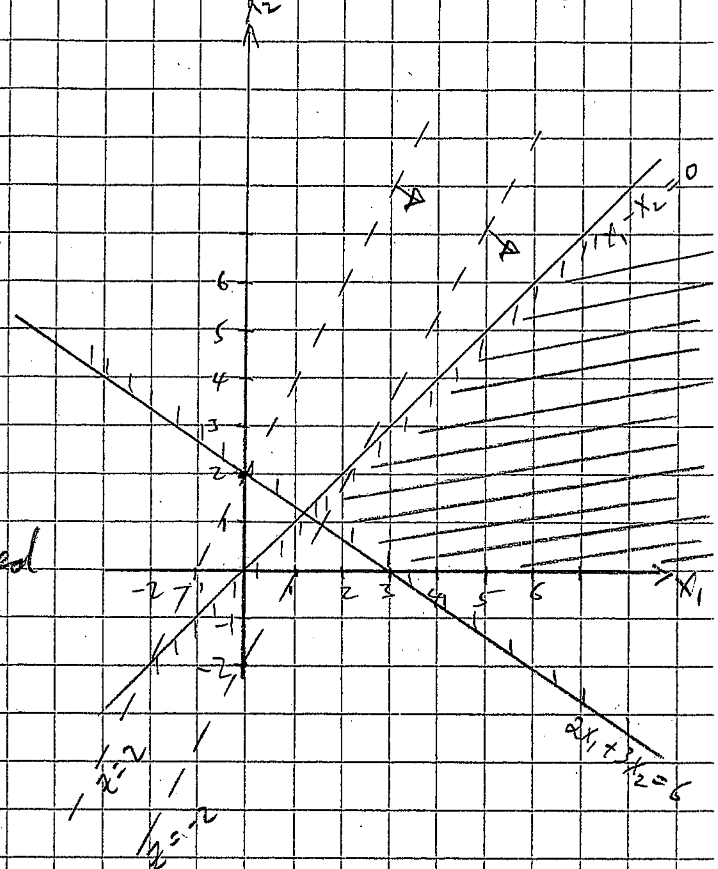
The LP is infeasible (no feasible solution), $\therefore$ no optimal solution

Extra

Solve the LP:

$$\min \quad z = -2x_1 + x_2$$

subject to
$$2x_1 + 3x_2 \geq 6$$
$$x_1 - x_2 \geq 0$$
$$x_1, x_2 \geq 0$$



Since the region is unbounded graph 2 z-lines

If $z = -2$, $-2x_1 + x_2 = -2$

If $z = 2$, $-2x_1 + x_2 = 2$

The z-line is moving towards the unbounded side of the feasible region, the z-value is unbounded, so no optimal solution.