MATH 111

HW #5

(1)

Ex 2.6

72/

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 70 & 62 & 113 & 93 & 51 & 29 & 54 & 19 & 82 & 67 & 116 \\ 39 & 41 & 69 & 57 & 28 & 15 & 33 & 9 & 40 & 43 & 71 \end{bmatrix}$$

$$\begin{bmatrix} 23 & 1 & 19 & 15 & 18 & 13 & 9 & 1 & 4 & 5 & 19 \\ 8 & 20 & 25 & 21 & 5 & 14 & 12 & 4 & 18 & 19 & 26 \end{bmatrix}$$

The decoded message is

"WHATS YOUR EMAIL ADDRESS?"

> *with*(*LinearAlgebra*) :

Ex. 2.7 (Economics) Question # 5

(2)

The input-output matrix is

> $A := << \frac{50}{180}, \frac{20}{180}, \frac{30}{180} > | < \frac{20}{160}, \frac{30}{160}, \frac{20}{160} > | < \frac{40}{120}, \frac{20}{120}, \frac{20}{120} >>;$

$$A := \begin{bmatrix} \frac{5}{18} & \frac{1}{8} & \frac{1}{3} \\ \frac{1}{9} & \frac{3}{16} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{8} & \frac{1}{6} \end{bmatrix} \quad (1)$$

The demand matrix is

> $Dmatrix := < 80, 90, 60 >;$

$$Dmatrix := \begin{bmatrix} 80 \\ 90 \\ 60 \end{bmatrix} \quad (2)$$

The output vector X is

> $Y := (IdentityMatrix(3) - A)^{-1};$

$$Y := \begin{bmatrix} \frac{162}{101} & \frac{36}{101} & \frac{72}{101} \\ \frac{208}{707} & \frac{944}{707} & \frac{272}{707} \\ \frac{258}{707} & \frac{192}{707} & \frac{990}{707} \end{bmatrix} = (I - A)^{-1} \quad (3)$$

> $X := Multiply(Y, Dmatrix); evalf(X);$

$$X := \begin{bmatrix} \frac{20520}{101} \\ \frac{117920}{707} \\ \frac{97320}{707} \end{bmatrix}$$

$$(I-A)^{-1}D = \begin{bmatrix} 203.1683168 \\ 166.7892504 \\ 137.6520509 \end{bmatrix} \quad (4)$$

Ex 2.7 Application (Economics)

9/ Each unit of R requires $^{30}/_{130}$ units of R
and $^{20}/_{130}$ units of S
Each unit of S requires $^{40}/_{70}$ units of R
$^{10}/_{70}$ units of S

Let x_1 be the level of production of R
x_2 — — — — — of S

then
$$\begin{cases} x_1 = \frac{30}{130}x_1 + \frac{40}{70}x_2 + 80 \\ x_2 = \frac{20}{130}x_1 + \frac{10}{70}x_2 + 40 \end{cases}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 30/130 & 40/70 \\ 20/130 & 10/70 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 80 \\ 40 \end{bmatrix}$$

$$\therefore I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3/13 & 4/7 \\ 2/13 & 1/7 \end{bmatrix} = \begin{bmatrix} 10/13 & -4/7 \\ -2/13 & 6/7 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 10/13 & -4/7 & 1 & 0 \\ -2/13 & 6/7 & 0 & 1 \end{array}\right]$$

$$\therefore (I-A)^{-1} = \begin{bmatrix} 3/2 & 1 \\ 7/26 & 35/26 \end{bmatrix}$$

$R1 = \frac{13}{10}r_1$
$$\left[\begin{array}{cc|cc} 1 & -26/35 & 13/10 & 0 \\ -2/13 & 6/7 & 0 & 1 \end{array}\right]$$

$$X = \begin{bmatrix} 3/2 & 1 \\ 7/26 & 35/26 \end{bmatrix} \begin{bmatrix} 80 \\ 40 \end{bmatrix}$$

$R2 = \frac{2}{13}r_2 + r_2$
$$\left[\begin{array}{cc|cc} 1 & -26/35 & 13/10 & 0 \\ 0 & 26/35 & 1/5 & 1 \end{array}\right]$$

$$= \begin{bmatrix} 160 \\ 980/13 \end{bmatrix}$$

$R2 = \frac{35}{26}r_2$
$$\left[\begin{array}{cc|cc} 1 & -26/35 & 13/10 & 0 \\ 0 & 1 & 7/26 & 35/26 \end{array}\right]$$

$\therefore x_1 = 160$ units

$R1 = \frac{26}{35}r_2 + r_1$
$$\left[\begin{array}{cc|cc} 1 & 0 & 3/2 & 1 \\ 0 & 1 & 7/26 & 35/26 \end{array}\right]$$

$x_2 = 980/13$ units

Heilbron

Ex 2.7 : Application (Statistics)

$$A = \begin{bmatrix} 5 & 1 & 1 \\ 2 & 3 & -1 \\ -1 & 6 & 2 \end{bmatrix}$$

Ex 2.7 : Application (Statistics)

(5)

10 /

(a) $X = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 4 \\ 1 & 6 \\ 1 & 8 \end{bmatrix}$ $\quad X^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & 6 & 8 \end{bmatrix}$ $\quad Y = \begin{bmatrix} 50 \\ 74 \\ 85 \\ 90 \\ 92 \end{bmatrix}$

$X^TX = \begin{bmatrix} 5 & 20 \\ 20 & 120 \end{bmatrix}$ $\qquad \left[\begin{array}{cc|cc} 5 & 20 & 1 & 0 \\ 20 & 120 & 0 & 1 \end{array}\right]$

$(X^TX)^{-1} = \begin{bmatrix} 3/5 & -1/10 \\ -1/10 & 1/40 \end{bmatrix}$ $\qquad R1 = \frac{1}{5}r_1 \left[\begin{array}{cc|cc} 1 & 4 & 1/5 & 0 \\ 20 & 120 & 0 & 1 \end{array}\right]$

$X^TY = \begin{bmatrix} 391 \\ 1764 \end{bmatrix}$ $\qquad R2 = -20r_1 + r_2 \left[\begin{array}{cc|cc} 1 & 4 & 1/5 & 0 \\ 0 & 40 & -4 & 1 \end{array}\right]$

$R2 = \frac{1}{40}r_2 \left[\begin{array}{cc|cc} 1 & 4 & 1/5 & 0 \\ 0 & 1 & -1/10 & 1/40 \end{array}\right]$

$A = \begin{bmatrix} 3/5 & -1/10 \\ -1/10 & 1/40 \end{bmatrix} \begin{bmatrix} 391 \\ 1764 \end{bmatrix}$ $\qquad R1 = -4r_2 + r_1 \left[\begin{array}{cc|cc} 1 & 0 & 3/5 & -1/10 \\ 0 & 1 & -1/10 & 1/40 \end{array}\right]$

$= \begin{bmatrix} 58.2 \\ 5 \end{bmatrix}$

The least squares line is $y = 58.2 + 5x$

(b) If $x = 9$ then $y = 58.2 + 5(9)$

$= 103.2$