Ex 2.6

10/.
$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the product of the two matrices is I, they are inverses of each other

12/.
$$\left[\begin{array}{cc|cc} 4 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$$

$R1 = \frac{1}{4} r1$
$$\left[\begin{array}{cc|cc} 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ 3 & 1 & 0 & 1 \end{array}\right]$$

$R2 = -3r1 + r2$
$$\left[\begin{array}{cc|cc} 1 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & -\frac{3}{4} & 1 \end{array}\right]$$

$R1 = -r2 + r1$
$R2 = 4r2$
$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & -3 & 4 \end{array}\right] \qquad \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \underline{\underline{\begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}}}$$

20/ see next page

26/
$$\left[\begin{array}{cc|cc} -1 & 2 & 1 & 0 \\ 3 & -6 & 0 & 1 \end{array}\right]$$

$R1 = -r1$
$R2 = 3r1 + r2$
$$\left[\begin{array}{cc|cc} 1 & -2 & -1 & 0 \\ 0 & 0 & 3 & 1 \end{array}\right]$$

Since R2 has all zeros on the left, it is impossible to obtain I on the left hand side. This matrix has no inverse

Ex 2.6

~~6/~~

$$\cancel{\begin{bmatrix} 1 & 5 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{5} & -\frac{1}{10} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

~~Since the product of the two matrices is the identity matrix, they are inverses of each other~~

20/

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R2 = -2r_1 + r_2$

$R3 = -r_1 + r_3$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R2 = -r_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R1 = -r_2 + r_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R2 = -r_3 + r_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 3 & -1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

The inverse is $\begin{bmatrix} -1 & 1 & 0 \\ 3 & -1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

Ex 2.6

48/ $\begin{cases} 3x + 7y = -2 \\ 2x + 5y = 10 \end{cases}$

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \qquad \left[\begin{array}{cc|cc} 3 & 7 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \qquad R1 = \tfrac{1}{3} r_1 \left[\begin{array}{cc|cc} 1 & 7/3 & 1/3 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$$

$$R2 = -2r_1 + r_2 \left[\begin{array}{cc|cc} 1 & 7/3 & 1/3 & 0 \\ 0 & 1/3 & -2/3 & 1 \end{array}\right]$$

$$X = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ 10 \end{bmatrix}$$

$$R2 = 3r_2 \left[\begin{array}{cc|cc} 1 & 7/3 & 1/3 & 0 \\ 0 & 1 & -2 & 3 \end{array}\right]$$

$$= \begin{bmatrix} -80 \\ 34 \end{bmatrix}$$

$$R1 = -\tfrac{7}{3} r_2 + r_1 \left[\begin{array}{cc|cc} 1 & 0 & 5 & -7 \\ 0 & 1 & -2 & 3 \end{array}\right]$$

$\therefore$ $x = -80$, $y = 34$