Abstract:

Project title:
**New Models for Disease Outbreaks Via Delay Differential Equations**

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Research abstract

The recent SARS epidemic has drawn attention back to the classical epidemic models. We study some natural extensions to include such aspects as an exposed period, and the reduction of contacts by susceptibles in response to an epidemic. Throughout the course of history, diseases have developed suddenly and then disappeared just as suddenly without attacking the entire community.

Most Recent Significant Contributions to Research (funded by Malaspina Research Grant, 2003)

Harvesting Strategies Using Differential Equations with Delay

The last decade has seen an expanding interest in problems involving non-linear differential equations with delay. The application of delay equations to biomodelling is in many cases associated with studies of dynamical phenomena like oscillations, bifurcations, and chaotic behaviour.

The majority of the models of a marine population (e.g., with harvesting strategies) in the literature ignore the fact that to determine, for example, the hunting quota, we do not know the population at the exact time; there is always a delay in processing and distributing field information. It is important to be able to model such reality for populations affected by some kind of delayed interference.

In [1-3] we considered a logistic-like differential equation with independent delays in the logistic (self-regulatory mechanism) term and in harvesting term. We obtained the following results:
a) A priori upper bound of solutions and explicit conditions of global stability for these equations.
b) Sufficient conditions for the existence of a non-oscillatory solution, for the oscillation of all solutions, and for the convergence of every non-oscillatory solution to equilibrium.

The results of this research have been accepted for publication in 2003, and some of the results are just submitted for publication in 2003. However, we anticipate that our results will be used in future research projects in the areas of Population Dynamics such as Fisheries, Forestry Resource Management, etc.

There are other evidences of impact and significance of this research:
1. The theoretical results of that type for the delay logistic equation with the delayed harvesting strategies have never been stated before.
2. The technique used for the investigation of stability and oscillation properties of the models of that type, could be used for further studies of more general delay differential equations of Population Dynamics.
3. We also found that the traditional a logistic-like model in some cases produces artificially complex dynamics. Our work indicates that in order to capture the oscillatory behaviour as in nature, it would be reasonable to get away from the specific logistic form in studying Population Dynamics and use more general classes of growth models.

References (contributions)


Potential impact of the research

1. Has potential to attract external funding, e.g., NSERC Discovery Grant (in progress)
2. Enhance the reputation of MUC through the international conferences, presentations on academic seminars (in Canada: UBC, SFU, University of Calgary; in Israel: University of Negev; in USA: American Institute of Mathematical Science)
3. Involves collaboration with MUC (Fishery and Biological Departments), and outside MUC (Norway, Calgary, Vancouver, Israel)
4. Leads to a tangible outcome: articles in scientific journals, international conferences proceedings.
5. Has potential for high impact on solving real world problems.
6. Has potential to involve undergraduate and postgraduate students into scientific research
7. The International Conference on Differential Equations and Applications in Mathematical Biology will be held first time at Malaspina University-College in 2004
8. (Member of Local Organizing Committee Dr. L. Idels) In cooperation with the University of Alberta, University of Calgary and Pacific Institute for Mathematical Science (Canada) this conference will attract more than 150 scientists all over the world.

RESEARCH PROPOSAL

RESEARCH OBJECTIVES AND MERIT OF THE PROPOSAL

The broad goal of this work

a) Develop new mathematical models of SARS outbreaks
b) Create a network of specialists in Mathematical Modeling (International Conferences, Workshops, and Collaboration)
c) Train HQP in Population Dynamics; help undergraduate, graduate and post-graduate students to start a research program

In the limited scope of the present proposal short-term objectives are focused on:

a) Introduction of more general Mackey-Glass model. The aim is to capture the abrupted behaviour often observed in the infected populations
b) Theoretical studies of the qualitative behaviour of the mathematical models, interpretation of our mathematical findings and possible field implementation of the developed epidemic models
c) Testing hypothesis that the negative feedback term with small delays helps to stabilize the outbreaks in the populations
d) Collecting and analyzing statistical data
e) Give a start up for undergraduate students involved in Biology Programs.
OUTLINE OF THE PROPOSED RESEARCH PROGRAM

An understanding of biological phenomena implies the ability to predict and control them. In this respect recent experience in the systematic refinement of mathematical models for HIV infection seems to be quite remarkable.

We shall introduce feasible types of delay epidemic models in order to understand the system dynamics.

Epidemic models are built upon various generalizations of the Mackey-Glass equation. That equation can be extended naturally to describe the dynamics of a controlled population.

Consider the general scalar non-linear differential equation with multiple delays and the delayed functional response

\[
dN(t) = rN(t-h(t))R[N(t-h(t))]-\alpha[N(t-g(t))]
\]

where \( r > 0, \ h(t) \leq t, \ g(t) \leq t, \ \alpha \geq 0, \ N(t) \) is the population size at time \( t \), \( R[N(t)] \) is the per capita net reproductive rate, \( F[N(t-g(t))] \) is a delayed functional response, \( h(t) \) is the developmental (maturation) time, \( g(t) \) is the duration of the infectious period or the immune period.

This differential equation incorporates a broad class of epidemic models \([1-4]\). All models (1) are united by the common theme of possessing a regime of infectious dynamic behaviour with control.

For example, if \( R[N(t-\tau)] = \frac{1}{1+[N(t-\tau)]^\gamma} \), \( \alpha = 0 \) and \( \gamma > 0 \),

then equation (1) is the Mackey-Glass equation with delay which is still widely used in the Biology and Medicine \([5-9]\), probably because of its biological clarity.

1) We will consider the following Generalized Mackey-Glass Delay Equation with Delayed Functional Response

\[
dN = \frac{rN(t-h(t))}{1+(\frac{N(t-h(t))}{K})^\gamma} - \alpha F[N(t-g(t))]
\]

Where \( \alpha > 0, \ \gamma > 0, \ r > 0, \ K > 0, \ h(t) \leq t, \ g(t) \leq t \)
2) Our results recently obtained for a logistic-like differential equation with delay, suggest the hypothesis that the negative feedback term with small delays helps to stabilize the population outbreaks. We will test this hypothesis for equation (2).

3) For equation (2) sometimes even the existence of a positive solution is an open problem [1−3].

We will obtain sufficient conditions for existence, positiveness, boundedness and extinction of the solutions of (2).

4) We will consider the following forms of the functional response:

(a)  \( F[N(t)] = N(t) \) (Linear model)

(b)  \( F[N(t)] = \frac{N(t)}{1 + N(t)} \) (Non-linear Hill’s model)

(c)  \( F[N(t)] = \frac{N^2(t)}{1 + N^2(t)} \) (Non-linear Holling’s model)

(d)  \( F[N(t)] = \exp[-N(t)] \) (Exponential model)

We will obtain sufficient conditions for existence and stability of the solutions of (2), using forms (a)-(d) with delayed argument.

5) It is a well-known fact [1, 2, 5, and 8] that continuous changes in the functional responses can cause discontinuous collapse in the population; continuous changes in environmental parameters can lead to discontinuous outbreaks of the population. The existence of an outbreak holds important implications for controlled spread of decease in community.

We will study the major effect of delays in the epidemic models. The characteristic of infectious dynamics being examined are the existence and stability of the steady states or the appearance of the abrupted solutions.

Methods of investigation

Qualitative methods: differential and integral inequalities, integral transforms, series expansions w. r. t. small parameters, asymptotic methods for delay differential equations.

Numerical methods: modifications of the Runge-Kutta method combined with the interpolation technique, adaptive grids used to adjust the method to impulsive models.

References (for a new application)

**Timelines:**

Jan 2004-April 2004: to analyse and develop delay models of the generalized Mackey-Glass model, develop and mathematically justify a new methodology of studying non-linear delay models, based on the generalized Mackey-Glass equation

May 2004-June 2004: to develop the appropriate software for the whole project.

June 12-18 Presentation on the Fifth AIMS International Conference (USA)*

July 3-24 Joint work with collaborators from Norway and Calgary at MUC**

June 2004-August 2004: The statistical data will be collected and analyzed. We anticipate at least three students of MUC to be involved in the summer session**

Sep 2004-Dec 2004: to perform numerical simulations, qualitative analysis of some non-linear equations, obtain some exact estimates of practical interest; to finish joint papers.

December 2004: to visit the collaborator (Israel)