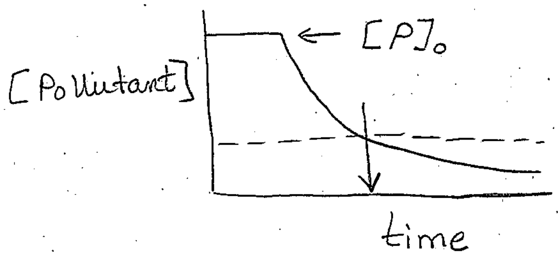
Examples:

1. The residence time of Okanagan Lake is approximately 60 yrs. How long would it take from the cessation of a discharge of a persistent contaminant to fall to 10% of its original concentration?

[Ans: 140 yr]

$$\tau_w = \tau_p = 60 \text{ yrs} \quad \therefore \; k = \frac{1}{60 \text{yr}} = 1.7 \times 10^{-2}\, \text{yr}^{-1}$$



$\leftarrow [P]_t = 0.10\,[P]_0$

must use integrated rate equation

$$[P]_t = [P]_0\, e^{-kt}$$

$$0.10\,[P]_0 = [P]_0\, e^{-(1.7\times10^{-2}\text{yr}^{-1})t}$$

$$\frac{0.10\,[P]_0}{[P]_0} = e^{-(1.7\times10^{-2}\text{yr}^{-1})t}$$

$$\frac{\ln(0.10)}{-1.7\times10^{-2}\text{yr}^{-1}} = t$$

$$\therefore t = 135 \text{ yr} \approx 140 \text{ yr}$$

2. If the concentration of a pollutant in Lake Superior were 1.0 ppm today, how many half lives would it take for its concentration to fall to 50 ppb ~~ppm~~ if all input of the pollutant into the lake ceased immediately?

[Ans: 4.3]

$$\frac{[P]_t}{[P]_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{50 \text{ ppb}}{1000 \text{ ppb}} = \left(\frac{1}{2}\right)^n$$

$$0.050 = \left(\frac{1}{2}\right)^n$$

$$\log(0.050) = n \log\left(\frac{1}{2}\right)$$

$$n = \frac{\log(0.050)}{\log 0.50} = \frac{-1.30}{-0.301} = 4.3 \text{ yrs}$$

3. A compound has a chemical degradation rate constant of $6.6 \times 10^{-3}\ s^{-1}$ in a 2500 m^3 water treatment plant. At what flow rate could the treatment plant be operated if a 10 minute water contact time is required to remove this compound?

[Ans: $1.8 \times 10^6\ m^3\ day^{-1}$]

$\tau_w = 10$ mins

$k_w = 0.10\ min^{-1}$

$k_d = 6.6 \times 10^{-3}\ s^{-1} \times \frac{60\ s}{1\ min} = 0.396\ min^{-1}$

$$\tau_p = \frac{1}{\Sigma_i k} = \frac{1}{0.496\ min^{-1}} = 2.02\ min$$

$$\tau_p = \frac{[pollutant] \cdot 2500\ m^3}{[pollutant] \cdot flow} = \frac{2500\ m^3}{flow}$$

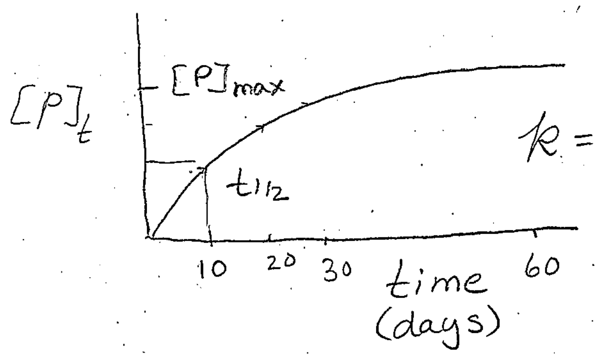
Since $\tau_p = 2.02$ min

$$\therefore flow = \frac{2500\ m^3}{2.02\ min} = 1.23 \times 10^3\ \frac{m^3}{min} \times \frac{60\ min}{hr} \times \frac{24\ hr}{day}$$

$$= 1.8 \times 10^6\ m^3/day$$

4. A soluble pollutant is dumped into a lake starting on day zero. The rate constant of the increase is $0.069\ day^{-1}$. The integrated rate equation is given by $[C]_t/[C]_{max} = (1 - e^{-kt})$. Sketch a labeled plot of the relative concentration over the first 60 days. What fraction of the steady state concentration is reached after 35 days?

[Ans: 0.91]



$k = 0.069\ day^{-1}$ $\quad \therefore t_{1/2} = \frac{0.693}{k} = 10$ day

$$\frac{[P]_t}{[P]_{max}} = \left[1 - e^{-(0.069\ d^{-1})(35 d)}\right] = \left(1 - e^{-2.41}\right)$$

$= 0.91$

In other words, the conc of pollutant reaches 91% of its steady state value after 35 days.