

Question 1:

Compute the determinant

$$\begin{vmatrix} 3 & 2 & -1 & 2 \\ 1 & 0 & -1 & 2 \\ 2 & 2 & -1 & 0 \\ 3 & 2 & -1 & 2 \end{vmatrix}.$$

Question 2

Prove (disprove) that

$$|A + B| \neq |A| + |B|.$$

Hint: use two matrices

$$\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$$

and

$$\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}.$$

Question 3:

Let $\bar{u} = (1, -1, 1)$ and $\bar{v} = (2, 1, -1)$.

1. Compute

$$(\bar{u} + \bar{v}) \cdot (\bar{u} - \bar{v})$$

2. Find the angle between two vectors \bar{u} and \bar{v} .

Question 4

Use Cramer's Rule to solve the system

$$\begin{aligned} x_1 - x_2 - 2x_3 &= 1 \\ x_1 - 2x_3 &= 2 \\ x_2 - 2x_3 &= 0. \end{aligned}$$

Question 5:

Find A^{-1} by using two methods: the Cofactor Formula and Gauss-Jourdan algorithm.

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix}.$$

Question 6

Let $\bar{u} = (3, -1, 0)$, $\bar{v} = (4, 0, 1)$ and $\bar{x} = (-2, 2, 1)$. Find two scalars α and β such that

$$\alpha\bar{u} + \beta\bar{v} = \bar{x}.$$

Question 7

Find such scalars a and b that A is a singular matrix

$$A = \begin{bmatrix} 1 & 2 & a \\ a & 1 & 0 \\ -1 & -2 & b \end{bmatrix}.$$

Question 8

Given three vectors \bar{u} , \bar{v} , \bar{w} and a scalar α . Decide which operation is defined:

1. $\bar{u} \cdot \bar{v} \cdot \bar{w}$

2. $\bar{u} \cdot \alpha$

3. $\|\bar{u} \cdot \bar{v}\|$

4. $\|\bar{u} + \bar{v}\|$.
