

# Math 251 Sample Test 3 – April 2016

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## Question 1.

A spring with spring constant  $4\text{N/m}$  is attached to a  $1\text{ kg}$  mass with friction constant  $5\text{Ns/m}$ . If the mass is initially displaced to the right of equilibrium by  $0.1\text{ m}$  and has an initial velocity of  $1\text{ m/s}$  toward equilibrium.

- (a) Determine if the mass passes through the equilibrium position, if so determine when it does so. (b) Determine if the displacement has any local extrema for  $t > 0$ .

Question 2.

Solve

$$xy'' = y' + (y')^3.$$

**Question 3.**

Solve the system

$$\begin{cases} \frac{dx}{dt} = 3x + y + t - 1 \\ \frac{dy}{dt} = -2x - e^t \end{cases}$$

**Question 4.** Use the Laplace transformation to solve the IVP

$$y' + 2y = f(t),$$

$$y(0) = 0 \text{ and}$$

$$f(t) = \begin{cases} 1 & \text{if } t \in [0, 1) \\ -1 & \text{if } t \in [1, \infty). \end{cases}$$

**Question 5.**

Consider BVP

$$y'' + a^2y = 0, y(0) = 0, y(\pi) = 1.$$

Determine values of  $a$  so that the BVP has: a) one solution, b) no solutions c) infinitely many solutions.