

Question 1:

Solve (i)

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}, \quad y(1) = 1.$$

(ii)

$$xy \frac{dy}{dx} = 2x - y^2.$$

(iii)

$$(1 - \ln x)dy = \left(1 + \ln x + \frac{y}{x}\right)dx.$$

(iv)

$$\frac{dx}{dt} = (x + t + 3)^2.$$

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**Question 2**

The population of a species of elk in Canada has been monitored for some years. When the population was 600, the relative birth rate was found to be 35 percent and the relative death rate was 15 percent. As the population grew to 800, the corresponding figures were 30 percent and 20 percent. Write a differential equation to model the population as a function of time, assuming that relative growth rate is a linear function of population.

**Question 3** Without solving the DE

[ points]

$$\frac{dy}{dt} = ay + by^2$$

$a > 0$  and  $b > 0$ , a) Find the equilibria. b) Graph the solution curves for :  $y(0) = 0$ ,  $y(0) = 1$ ,  $y(0) = -a/2b$ , and  $y(0) = -a$ .

Question 4

Problem 1.

When a cold drink is taken from a fridge its temperature is 5 degrees C. After 25 minutes in a 20 degree C room, its temp increased to 10 degrees C. what the temperature of the drink after 50 minutes. When will the temperature be 15 degrees C?

Problem 2.

Suppose the population of wolves in a national park grows according to the logistic differential equation

$$\frac{dP}{dt} = 0.01P(100 - P).$$

a) If  $P(0) = 20$ , solve for  $P$  as a function of  $t$ .

Use your answer to find  $P$  when  $t = 3$  years. Use your answer to find  $t$  when  $P(t) = 80$  animals.

Which of the following statements are true?

1. The growth rate of the wolf population is greatest when  $P = 50$ .
2. If  $P > 100$ , the population of wolves is increasing.

Problem 3.

Consider a tank with 200 liters of salt-water solution, 30 grams of which is salt. Pouring into the tank is a brine solution at a rate of 4 liters/minute and with a concentration of 1 grams per liter. The well-mixed solution pours out at a rate of 5 liters/minute. Find the amount at time  $t$ .

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