MALASPINA UNIVERSITY-COLLEGE DEPARTMENT OF MATHEMATICS

FINAL EXAM: MATH 340 "APPLICATIONS OF MATHEMATICS"

December 8 2005

NAME: (print surname first)——————————————

STUDENT NUMBER——————————————-

SIGNATURE——————————————

INSTRUCTIONS:

Show all your work. Read each question carefully. Pace yourself. Notes and calculators are allowed. This exam consists of 6 questions out of a total of 90 marks. You are expected to supply complete solutions to all of the problems. Guessing the answer, even if right, shall carry no credit.

I have truly enjoyed being your Applied Mathematics professor this tough mathematics course.
You are amazing students : smart, intelligent, and hardworking!
Thank you very much for all the fun, good luck, and Happy New Year 2006
GOODBYE MODELVILLE!
Instructor Lev V. Idels

Question #1(15pt) (Lotka-Volterra Model, Modeling with a System of ODE)

Let P(t) denote the size at time t of a parasite population that preys on a host population of size H(t).

The competition model is described as follows

$$\boxed{\frac{dP}{dt} = (a - bP)H}$$
$$\boxed{\frac{dH}{dt} = (cH - d)P} \quad (1)$$

assume a, b, c, and d are positive constants

(a) Explain the significance of each factor in the right-hand sides of (1)

(b) Find all equilibrium points of (1)

(c) Show that if $P(t) > a/b$, then P(t) decreases, and that if $H(t) < d/c$, then H(t) decreases. Describe the implications of these results for the host-parasite populations

Question#2(10pt) (Discrete Optimization)

Certain lab animals must have at least 30 grams of protein and at least 20 grams of fat per day. Food A costs 18 cents per unit and each unit supplies 2 grams of protein and 4 grams of fat. Food B costs 12 cents per unit and each unit supplies 6 grams of protein and 2 grams of fat. Food B is bought under a long-term contract which requires that at least 2 units of B be used per day. How many units of each food must be used to minimize the daily cost?

Question #3 (10pt)(Continuous Optimization)

Three alleles (alternative versions of a gene) A, B, and O determine the four blood types A (AA or AO), B (BB or BO), O (OO), and AB. The Hardy-Weinberg Law states that the proportion of individuals in a population who carry two different alleles is $P = 2pq + 2pr + 2rq$

where p, q, and r are the proportions of A,B, and O in the population. Use the fact that $p + q + r = 1$ to show that P is at most 2/3.

Question#4(20pt) (Difference Equations. The Spread of a Contagious Disease)

Suppose there are 400 students in a college dormitory and that one or more students has a severe case of the flu. Let represent the number of infected students after n time periods. Assume that some interaction between those infected and those not infected is required to pass on the disease. If all are susceptible to the disease, represents those susceptible but not yet infected after n time periods. If those infected remain contagious, we may model the change in infecteds as proportional to the product of those infected by those susceptible but not yet infected.

a) Construct a difference equation to model the spread of a contagious disease.

b) There are many refinements to this model. Discuss them.

c) The more sophisticated models treat the infected and susceptible populations separately.

Construct a difference equation for that model.

Question #5(15pt) (Gradient Search Method)

Consider the objective function $f(x, y) = 0.5(x^2 + 2xy + 2y^2) - 3x - 2y + 6$, and begin the search at the point(2,1), find the minimum point for the objective function.

Question#6(20pt) (Modeling with ODE)

<u>Write a DE</u> that is a mathematical model of the situation described below:

(i) The acceleration of a Ferrari is proportional to the difference between 250km/h and the velocity of this car

(ii) Suppose a certain lake is stocked with fish, the birth and death rates are both proportional to the fish population.

(iii) Consider an alligator population in a lake satisfying the logistic equation. Now suppose that because of hunting, alligators are removed from the lake at the rate proportional to the existing alligator population.

(iv) A 30-year old woman accepts an engineering position with a starting salary of \$45,000 per year. Her salary $S(t)$ increases exponentially, with $S(t) = 45e^{t/20}$ thousands \$ after t years. Meanwhile, 12% of her salary is deposited continuously in a retirement account, which accumulates interest

at a continuous interest rate of 6%. Derive the differential equation satisfied by the amount $A(t)$ in her retirement account after t years.