MATH 346
Assignment #3
Spring 08
Due: Friday, February 23, 2008

Problem 1. Complete problem 3.9.2 from Chapter 3 of the text

Problem 2.
A pest population x grows exponentially according to the equation $x' = x$. We want to eliminate the pest by removing it at a rate h. However, we cannot sustain the treatment as a constant rate h. Instead our treatment decreases exponentially over time according to the formula $h = me^{-t}$. Here $m > 0$ is the maximal treatment rate (applied initially). Suppose the pest population has size $x_0 > 0$ at the time $t_0 = 0$ when the treatment starts.

a) Write down a formula for the solution.

b) Use your solution formula in a) to show what the treatment is effective iff $m \geq 2x_0$.

Problem 3.
(Control by overcrowding and by fishing). The IVP $y' = y - \frac{y^2}{9} - \frac{8}{9}$, $y(0) = y_0$

a) What is the overcrowding coefficient?
What is the harvesting rate?

b) Find the two positive equilibria.

c) Graph solution curves for various values of y_0. Use the axis ranges $0 \leq t \leq 10$, $0 \leq y \leq 15$. Interpret what you see in terms of the future of the fish.

Problem 4.
In an inland Australian river system, a model for a native fish which feeds off algae is described by the equation

$$\frac{dp}{dt} = 0.08p\left(1-\frac{p}{1000}\right)\ \left(1-\frac{200}{p}\right) \quad (1)$$

a) Show that the population is increasing if $200 < p < 1000$, and decreasing if $0 < p < 200$

b) Find all equilibria of (1) and state which of the equilibria is stable.

c) Show that the population $p(t)$ is given by

$$p(t) = \frac{1000A + 200e^{-0.064t}}{A + e^{-0.064t}}$$

where $A = \dfrac{200 - p_0}{p_0 - 1000}$

d) Sketch the graph of $p(t)$ for each of the following cases

$p_0 = 250 \quad , \quad p_0 = 100 \quad , \quad p_0 = 1500$

Explain the significance of the case $p_0 = 100$.

Problem 5.
Models that are commonly used in Fishery are:

Gompertz model $\dfrac{dN}{dt} = rN \ \ ln\left(\dfrac{K}{N}\right)$

Food-Limited model $\dfrac{dN}{dt} = \dfrac{rN(K-N)}{K+\beta N}$

Analyze the behaviour of the solutions to these equations assuming $K, \beta, r > 0$, that is

a) Determine all possible equilibria.

b) Describe what happens to the populations after a very long time.

c) Discuss the stability questions.

Problem 5 cont.

d) What does your analysis suggest about the fate of the populations?

e) Graph solution curves for various values of parameters r, β, K .