

Point Sampling (a.k.a. prism cruising)

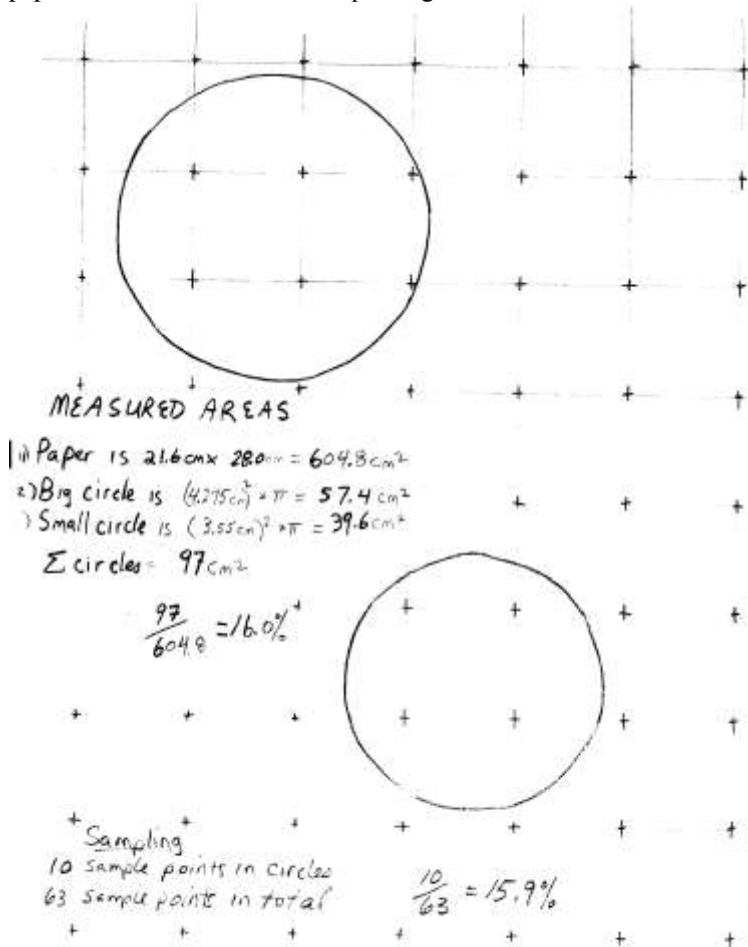
The following is a (simple?) explanation of the principles behind prism cruising. This is not meant as a stand alone paper; it is intended to supplement lecture/lab material given in class.

So how is it that counting trees “in” a prism plot gives us basal area per hectare?

PRINCIPLE FIRST (then the compilation)

Point number one: PROPORTION

This type of sampling gives us a proportion - visualize it as stump area. The image below is a scan of a paper with 2 circles drawn on it plus a grid of ‘tics’.



- The paper measures 604.8 cm^2 ($21.6\text{ cm} \times 28.0\text{ cm}$).
- The big circle measures 57.4 cm^2 ($(4.275\text{cm})^2 \times \pi$)
- The small circle measures 39.6 cm^2 ($(3.55\text{cm})^2 \times \pi$)
- % of paper in circles = 16.04% ($= 97\text{cm}^2 \div 604.8\text{ cm}^2$)

This value of 16% is the “true” measured proportion of area of the paper occupied by circles (i.e. stumps). Would we get a similar answer if we **sampled** instead of measuring? Let’s see...

- ⇒ The “dot grid” laid on the paper has a total of 63 dots (9 x 7)
- ⇒ There are 10 dots contained in circles.
- ⇒ % of dots in circles = 15.87% = $10 \div 63$ (10 “hits” out of 63 “attempts” or 10 of the 63 plots landed inside a stump)

If we establish plots in the field in some non-bias fashion (i.e. a randomly placed grid) we will get a proportion of the plots landing inside stumps (or inside trees if we were cruising standing timber). The proportion of hits is the proportion of the area taken up by stumps (i.e. 100 plots, 2 land in stumps ... therefore, 2% of the area is occupied by the stumps). This idea of **proportion** is the key!

To implement this let’s pretend you establish plots on a grid in a clearcut. All you have to record is whether or not the plot location was “inside a stump”. The number of plots that land “inside stumps” will be “hits”. The number of “hits” \div the total number of plots will give us proportion of the land base occupied by stumps. This is a measure of basal area (“BA” for short). Two minor difficulties here: efficiency and units.

Point number two: EFFICIENCY

Say the site has a basal area of 50m² / ha (i.e. of the 10,000 m² “land area” in a hectare, only 50 m² is “stump area”). If you did 10,000 plots, chances are that only 50 of these would land inside a stump (expressed as a percent this is 0.5%, or 0.005 as a proportion). That’s a lot of traversing and sample points to establish to get 50 “hits”.

So, what if we pretend that all the stumps are bigger than they really are. Let’s “blow them up” by say 900 times. This should increase efficiency. This means that if you did 10,000 plots, chances are that you would tally 45,000 stumps (900*50). Said another way, on average each plot would land inside the “blown-up area” of 4.5 stumps (these “blown-up stump areas” obviously overlap one another). Said one last time, “each plot has an average of 4.5 stumps in it”. Expanding the “stump area” certainly addresses efficiency. In fact we could now do far less plots (10,000 plots is ridiculous). In this example, if we did 10 plots we would likely sample 45 stumps. The proportion would still be the same:

$$\frac{45,000 \text{ stumps}}{10,000 \text{ plots}} = \frac{45 \text{ stumps}}{10 \text{ plots}} = \frac{4.5 \text{ stumps}}{\text{plot}}$$

We have to remember that our cruise will be now be getting a proportion that is 900 x’s too big. That’s OK, we’ll just divide by 900 later during the compilation stage. In this case,
 $4.5 \text{ (average number of stumps per plot)} \div 900 = 0.5\% (= 0.005 \text{ as a proportion})$

Notice this final answer (0.5%) from doing 10 plots with blown-up stumps is the same value as doing 10,000 plots without blowing-up the stumps (50 stumps \div 10,000 plots).

Point number three: UNITS

By convention we express **BA** in **m²/ha** - that is *m² of stump area* per hectare. What we have so far is a proportion (0.5%). It doesn’t really have any units. To convert this to m²/ha, all we do is multiply by 1. In our case 1 is **10,000 m² / ha** (since there are 10,000 m² in a hectare, this really is equivalent to 1).

$$0.5\% \text{ is really } 0.005, \text{ so } \dots 0.005 * 10,000 \text{ m}^2 / \text{ha} = 50 \text{ m}^2 / \text{ha}$$

SUMMARY of the Principle:

We are establishing a **proportion of BA** when we cruise with a prism. The prism **increases efficiency by “blowing up” trees**; each tree now has a very large imaginary circle around it ... “it’s very own plot area”. When we sample we stop at a point and count all the trees whose imaginary circles overlap our point ... these trees are “in our sample”. That’s all there is for the field part of getting BA - we do relatively few plots but get to count many “in” trees. In the compilation we have to **“blow down”** the number of hits (otherwise known as ÷ blowup factor) and then **convert from % to m²/ha**.

COMPILATION

This next section provides an overview of the calculations required to get BA and then, more importantly, volume.

First: BASAL AREA

This will basically be a repeat of the principle described previously. We can calculate *average* basal area of many plots with the following formula:

$$\begin{aligned} \text{BA} &= (\text{total “in trees”} \div \text{total plots}) \div \text{blowup factor} * 10,000 \text{ m}^2/\text{ha} \\ &= \text{ave. tree count} \div \text{blowup factor} * 10,000 \text{ m}^2/\text{ha} \end{aligned}$$

In our example, we had an average of 4.5 trees per plot (and remember this number is 900 x’s too big)

$$\text{BA} = 4.5 \text{ (average stumps/plot)} \div 900 \text{ (blow-up factor)} * 10,000 \text{ m}^2/\text{ha} = 50 \text{ m}^2/\text{ha}$$

Second: HEY, WE WANT VOLUME NOT BASAL AREA – VBAR

Details, details. OK, you asked for it. To get volume/ha from BA/ha we need a “fix it ratio” ... one that magically converts m²/ha → m³/ha.

Sampling for BA is cheap and easy (*that’s not to say that you’re cheap and easy, well ...*). If we could get a *volume/BA ratio* we’d be laughing (*perhaps hysterically in a padded room*). Say, what if every now and then we measure trees with diameters and heights and all that stuff. We could calculate the volume of each tree by plugging the tree measurements into volume equations. We could also calculate the basal area of each tree. Hence we can get a volume to basal area ratio for each tree.

Because **volume to basal area ratio** is such a mouthful, we use the term **VBAR** (Vol. to BA Ratio). VBAR can be visualized as the volume of wood stacked on top of a stump divided by the area of the stump.

If we multiply average BA (m²/ha) by average VBAR (m³/m²) the basal area (m²) cancels out and we are left with (m³/ha). Note that we use uppercase BA to denote basal area on a per hectare basis and lowercase ba to denote the basal area of a tree. The equation looks like this:

$$\begin{aligned} \text{Volume/ha} &= \frac{\text{BA}}{\text{ha}} * \frac{\text{volume}}{\text{ba}} \\ &= \text{BA (m}^2/\text{ha)} * \text{VBAR (m}^3/\text{m}^2) \end{aligned}$$

By the way, VBAR is fairly consistent whereas BA/ha can be quite variable. This is good because we just figured a way to get BA/ha cheaply – counting trees! Think of the brilliance here ...

- BA is the variable factor (hence we need lotsa plots to feel good), luckily it can be sampled quickly and cheaply ☺
- measuring trees is more time consuming so VBAR is more expensive to sample, BUT it is less variable (hence we measure relatively few trees and still feel good ☺).

SUMMARY SO FAR

What we really want is volume/ha. We know that BA/ha is closely related to volume/ha. Further to this we found a cheap way to sample BA/ha – counting trees. To convert BA/ha to volume/ha we need a conversion factor, namely VBAR. We get VBAR by measuring trees. *So the two ideas to grasp are:*

- *counting trees gives us BA/ha (done cheap & easy), and*
- *measuring trees gives us VBAR (which converts BA/ha to volume/ha).*

THE STATISTICS (ignore this short section if you haven't covered this in stats class yet)

Believe it or not, there is nothing new to us here. We have sampling error on count plots (BA/ha) and on the ratios (VBAR). We have to combine these errors. The equations is,

$$SE\% \text{ combined} = \sqrt{SE\%_{\text{counts}}^2 + SE\%_{\text{ratios}}^2}$$

This time we get to calculate with SE in % (no need to convert to units, since the units are different anyhow).

WHAT IS BAF

If you understood the first page on *PRINCIPLE* then you already understand BAF. BAF stands for **Basal Area Factor**. You use this number to multiply your tree count to get BA/ha. Remember the formula,

$$BA/ha = \text{tree count} \div \text{blowup factor} * 10,000 \text{ m}^2/\text{ha}$$

(this formula is the one to know as it shows the principle, all other equations are derived from here)

if we simplify the equation we get,

$$BA/ha = \text{tree count} * \frac{10,000 \text{ m}^2/\text{ha}}{\text{blowup factor}} = \text{tree count} * \text{BAF}$$

Thus,

$$\text{BAF} = \frac{10,000 \text{ m}^2/\text{ha}}{\text{blowup factor}}$$

A 20 BAF prism has a blowup factor of 500. A 10 BAF prism has a blowup factor of 1,000. Guess which prism size is more likely to be used in a stand with low basal area? ... the lower the basal area the less likely a tree will be in your plot ... so you'd want to "blow up" the trees a bit more to increase the chances ... so is that a BAF 20 or a BAF 10???

DERIVATION OF PRF

Now the more cumbersome part ☹ ... derivation of PRF. (Please, do not try this at home without mathematical supervision).

plot area = tree area * blow-up factor

*note that "plot area" = plot area of a tree (i.e. the imaginary circle), and that "tree area" can be thought of as "stump area",
now, we want **plot radius** (not plot area) so, remember the equation for area of a circle? ...*

$\pi * \text{plot radius}^2 = \pi * \text{tree radius}^2 * \text{blow-up factor}$

*but I want **plot radius** with no pi's or squaring (the only pi I want to see is at dessert), so,*

- *let's eat the pi on both sides, and*
- *square root both sides*

plot radius = tree radius * $\sqrt{\text{blow-up factor}}$

the prism in my pocket has BAF on it, not blow-up, so let's get BAF into the equation, remember
$$\text{BAF} = \frac{10,000 \text{ m}^2/\text{ha}}{\text{blowup factor}}$$

therefore,

$$\text{blow-up} = \frac{10,000 \text{ m}^2/\text{ha}}{\text{BAF}}$$

plot radius = tree radius * $\frac{\sqrt{10,000}}{\sqrt{\text{BAF}}}$

plot radius = tree radius * $\frac{100}{\sqrt{\text{BAF}}}$

we can get rid of the 100 now as plot radius is in m. and tree radius is in cm (100 x's too big), therefore

plot radius (m) = tree radius (cm) * $\frac{1}{\sqrt{\text{BAF}}}$

*last, we measure tree diameter not radius, so if we plug in diameter for convenience we also have to * 1/2*

plot radius(m) = tree diameter (cm) * $\frac{0.5}{\sqrt{\text{BAF}}}$

The PRF (plot radius factor) is the magic number we multiply tree diameter by to get its critical distance (i.e. whether it is in or out). PRF equals the last part of the equation,

$$\text{PRF} = \frac{0.5}{\sqrt{\text{BAF}}}$$

If we look at the equation for PRF we can reason it out:

- we utilize 0.5 (or 1/2) because we measure tree diameter, but it's tree radius that is proportional to plot radius,
- we have BAF and not blow-up because that is what is etched on the prism, and
- we have a $\sqrt{\quad}$ (square root) because the blow-up is based on **area** but we want plot **radius** (to convert to radius we did the pi r² thing, the pi's canceled out and we had to $\sqrt{\quad}$ the rest).

A worked example ** if you haven't come to compilations yet – ignore the rest ☺

Refer to the attached table. It has 4 sections:

1. raw data - from count (C) and full measure (M) plots,
2. by the tree - vol, ba and VBAR, on a per tree basis,
3. by the plot - plot calculations of BA, ave VBAR and VOL/HA,
4. at the bottom are the Volume/ha and stats.

Raw data

The raw data is hopefully straight forward: trees per plot, species and dbh & ht for measure plots.

By the tree

Tree volume was calculated based on that ugly formula for immature Fd, ba/tree is self-explanatory, VBAR is on a tree basis (vol/ba).

Plot

For **plot BA** it's simply the tree count * BAF. To get vol/ha we first need a conversion ratio (alias VBAR). For a plot basis we simply average the VBAR's of the trees in that plot.

Cruise Summary

At the bottom are the statistics. The **boldface** values show how the calculation is done for count **and** full measure plots.

$$\text{Volume} = \text{ave. BA/ha} * \text{ave. VBAR (vol/ba)} = 31.4 \text{ m}^2/\text{ha} * 12.9 \text{ m}^3/\text{m}^2 = 406 \text{ m}^3/\text{ha}$$

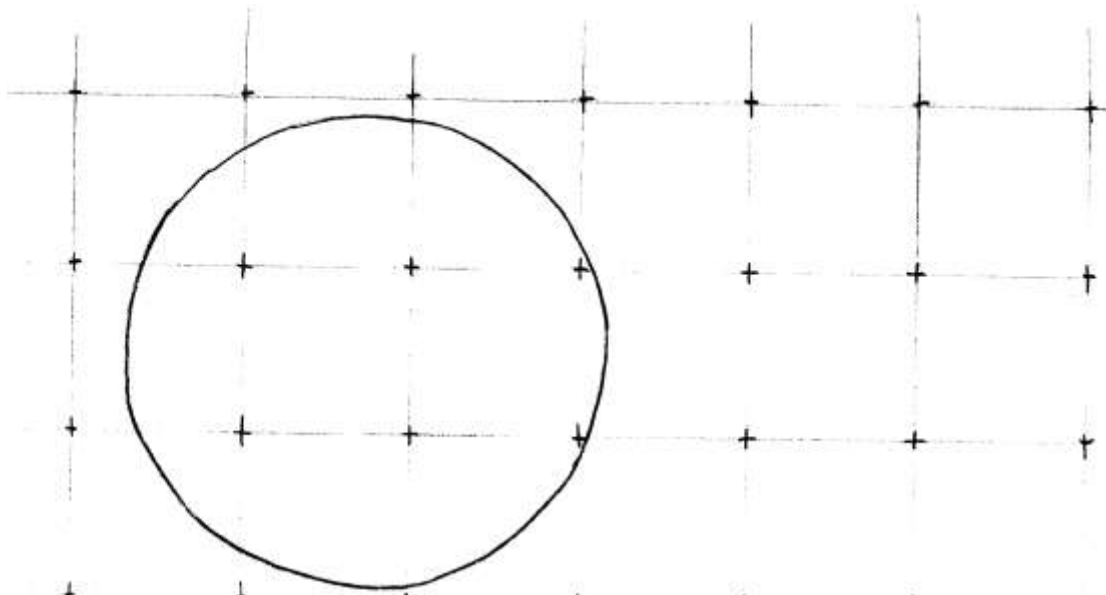
The *italic* values (also in gray shade) show the compilation as if no count plots were done. For interest sake, note the difference. Note also, that if you did a cruise with count plots you would only do the calculations shown in bold (there would be no need to do the italic - I just did it for comparison).

Volume = ave. of vol/ha
SE% is simply based on the variation of vol/ha between plots

The sampling errors (and standard errors) are high because of relatively few plots in the example.

BAF= 11

raw data							by the tree			PLOT		
plot	type	total trees	spp	dbh	ht	tree vol.	ba/tree	VBAR (tree)	BA	Full Measure Only		
										ave VBAR	VOL/HA	
1	C	3	FD						33			
2	M	3	FD						33	12.374	408	
				39	36	1.546	0.119	12.940				
				33	35	1.109	0.086	12.963				
				25	29	0.551	0.049	11.220				
3	C	4	FD						44			
4	M	3	FD						33	13.483	445	
				23	34	0.559	0.042	13.451				
				36	37	1.376	0.102	13.515				
				25	29	0.551	0.049	11.220				
5	C	2	FD						22			
6	M	2	FD						22	14.054	309	
				35	39	1.381	0.096	14.352				
				38	38	1.560	0.113	13.757				
7	C	3	FD						33			
						mean =	12.9	31.4		387.5		
						n =	8.0	7.0		3.0		
						sd =	1.1	7.6		70.2		
						cv% =	9%	24%		18%		
						se% =	3.1%	9.1%		10%		
						t-value =				4.303		
						e% =				45.0%		
						se% combined* =		9.7%				
						* based on ave. BA & ave VBAR						
						t-value =		2.365				
						e% @95% =		22.8%				
						vol (BA * VBAR) =		406				
										the calculations above would be for M plots only (eg no counts)		



MEASURED AREAS

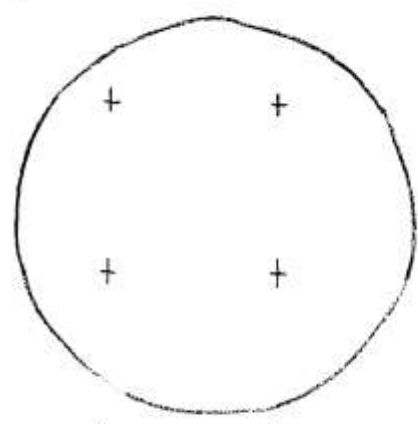
1) Paper is $21.6\text{cm} \times 28.0\text{cm} = 604.8\text{cm}^2$

2) Big circle is $(4.275\text{cm})^2 \times \pi = 57.4\text{cm}^2$

3) Small circle is $(3.55\text{cm})^2 \times \pi = 39.6\text{cm}^2$

Σ circles = 97cm^2

$$\frac{97}{604.8} = 16.0\%$$



Sampling
 10 sample points in circles
 63 sample points in total

$$\frac{10}{63} = 15.9\%$$