

More on Residence Times and Half-lives

The terms *residence time* and *half-life* are sometimes confused. To make matters worse, in some contexts (particularly chemical kinetics) the *residence time* is referred to as the *lifetime*.

Residence times can be determined for a reservoir itself (such as the water in a lake) or for a substance within it (such as a contaminant). The residence time for any reservoir can be determined by taking the ratio of the total amount (mass or volume) of the reservoir to the flux (expressed in units consistent with those in the numerator). Either the total rate of influx or outflux can be used, since for any system at steady state, the two are equal in magnitude.

$$\text{Residence time} = \frac{\text{amount of substance in the reservoir}}{\text{rate of influx (or outflux)}}$$

The *residence time (or lifetime)* of a substance within a reservoir may be governed not only by the residence time of the reservoir itself (essentially a dilution or flushing effect), but also by a variety of physical, chemical and biological processes. Sedimentation, degradation and microbial action often accelerate the removal of contaminant species. The residence time (lifetime) of a substance is determined with the knowledge of the amount (or concentration) of that substance and the combined rate of loss of the substance from the reservoir. Alternately, the residence time can be calculated as the reciprocal of the *sum of all first order rate constants*.

$$\text{Residence time of substance} = \frac{\text{amount of substance}}{\text{total of all removal rates}} = \frac{1}{\sum \text{removal rate constants}}$$

Although the residence time (or lifetime) is often easier to directly estimate than the half-life, the latter can be easier to interpret. It turns out that for first order processes, the half-life is roughly 70% of the lifetime as shown by the equations below.

Half-life:
$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{k}$$

Residence time:

$$\tau = \frac{1}{k}, \text{ where } k' \text{ represents the sum of all first order (or } pseudo\text{-first order) rate constants}$$

Therefore;

$$\text{Half-life} = 0.693 \times (\text{Residence time})$$

The *half-life* is the time required for the amount of substance to reach one half of its initial value.

Hence, to calculate the amount of material remaining after some time, apply the following relation;

$$C_t = C_o \left(\frac{1}{2}\right)^n$$

where C_t is the concentration after some time (t), C_o is the initial concentration and n is the number of half-lives. As you would expect, after one half-life the concentration will be $C_o \left(\frac{1}{2}\right)^1$ or one half of its original value. After two half-lives, the concentration will be $C_o \left(\frac{1}{2}\right)^2$ or one quarter of its original value and so on. This works equally well for non-integer values of n .

The half-life ($t_{1/2}$) is the amount of time required for a species to drop to one half of its original concentration, whereas the residence time (τ) is the time required for a species to drop to $1/e$ (i.e., $1/2.7 = 0.37$) of its original value. Strictly speaking, $t_{1/2}$ is about 70% of the residence time. However, this distinction is sometimes not made since both are of the same order of magnitude and there are generally large uncertainties in the estimation methods used to establish amounts and flow rates.

Examples:

1. The mass of nitrogen in the atmosphere is 4×10^{18} kg, and its sinks from the atmosphere include (i) biological nitrogen fixation by bacteria, 2×10^{11} kg yr⁻¹; (ii) production of NO in thunderstorms, 7×10^{10} yr⁻¹; (iii) chemical synthesis of ammonia, 5×10^{10} kg yr⁻¹ (all data refer to loss of nitrogen). Calculate the residence time of nitrogen in the atmosphere.

[Ans; $\tau(\text{atm } N_2) = 1.3 \times 10^7 \text{ yr}$]

2. The concentration of lead in the blood of an adult male was 140 micrograms per liter ($\mu\text{g/L}$), and the blood volume was 4.8 L. The net transfer of lead into this person's bones was 7.5 $\mu\text{g/day}$, and the net excretion rate was 24 $\mu\text{g/day}$. Calculate the residence time of lead in this person's blood.

[Ans; $\tau(\text{blood Pb}) = 21 \text{ days}$]